

# The Inevitability and Ubiquity of Cycling in All Feasible Legal Regimes: A Formal Proof

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## Abstract

Intransitive choices, or cycling, are generally held to be the mark of irrationality. When a set of rules engenders such choices, it is usually held to be irrational and in need of reform. In this article, we prove a series of theorems, demonstrating that all feasible legal regimes are going to be rife with cycling. Our first result, the Legal Cycling Theorem, shows that unless a legal system meets some extremely restrictive conditions, it will lead to cycling. The discussion that follows, along with our second result, the Combination Theorem, shows exactly why these conditions are almost impossible to meet. All of this has numerous implications, which we can here only allude to. For one, it suggests why law is as susceptible to manipulation and loophole exploitation as it has proved to be.

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# 1 Introduction

Cycling, or intransitivity, is generally viewed as the hallmark of irrationality. To make cyclical choices, the argument goes, is to be incoherent. In fact, many an argument in law is built on demonstrating that adopting certain rules leads to cycling. A fairly straightforward and mundane example of how basic the assumption of transitivity is might be a case like the following. Suppose a defendant is charged with negligence for his choice between two courses of action,  $x$  and  $z$ . Suppose he is able to point to two precedents, the first of which held that a defendant who chose  $x$  over  $y$  was not negligent and the second of which held that a defendant who chose  $y$  over  $z$  was not negligent as well. He will presumably feel on strong ground arguing that respect for precedent demands that in choosing  $x$  over  $z$ , he not be found negligent either. That only follows of course on the assumption of transitivity.<sup>1</sup> But the significance of transitivity runs deeper: When there is intransitivity, there is room for manipulation. If the law permitted a legal actor to choose  $a$  over  $b$  and  $b$  over  $c$ , it would be awkward, if it did not also permit him to choose  $a$  over  $c$ . If it allowed for such intransitivity, he might manipulate the law by changing the order of the choices. More generally, transitivity of choice is a central tenet both in economics and in law. Thus, a successful challenge to transitivity would open up all kinds of new questions. For example, in economics it might lead one to rethink something as basic as “revealed preferences,” namely how to infer preferences from choices. In law, it might lead one to rethink our customary approach to issues of form versus substance and the exploitation of loopholes. But that really only skims the surface of the arguments and models in both law and in economics in which intransitivity plays some role, and that might therefore require revisiting.

What we seek to demonstrate, with the help of two formal theorems, and a series of illustrations involving familiar legal systems, is that all remotely feasible legal systems, and certainly all that are known to have existed, are riddled with cycles. Nor are they mere occasional pathologies; they are rampant.

Our first theorem, the Legal Cycling Theorem, shows that all legal systems that are not what we call option-stratified, are going to exhibit cycles. We then present a number of examples and considerations to substantiate the claim that we would not want a legal system that is option-stratified. Our second theorem, the Combination Theorem, shows why an option-stratified system is almost impossible to construct, even if one wanted to go to the trouble of creating one.

Our results, as will become evident quickly, are rooted in social choice and therefore bear

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<sup>1</sup>Let us assume these precedents are Court of Appeals decisions, finding negligence absent “as a matter of law,” rather than factual determinations at the trial court level that would not have the required precedential significance.

an interesting, though not straightforward, relationship to Arrow’s famous impossibility theorem which has already had an important impact on law by way of a variety of seminal contributions such as Spitzer 1979; Easterbrook 1982; Kornhauser and Sager 1986; Miller and Rachmilevitch 2014. But there is one particularly obvious and notable difference between our work and much (but not all) of the prior work drawing on social choice, which is that we are not here concerned with collective decision-making.

This paper is organized as follows: The next section introduces the formal notation and shows a full characterization of the legal systems that induce cycles (the Legal Cycling Theorem.) The subsequent section proceeds to show that actual legal systems induce cycles. Section 4 shows that attempts to change laws and eliminate cycles either fail or have implications that are so perverse as to be unacceptable. Section 5 lays bare an alternative source of cycling in legal regimes, arising from the fact that combining legal doctrines in any plausible fashion often leads to cycling (the Combination Theorem). Section 6 explains the relationship of the foregoing to the previous social choice literature. Section 7 revisits some classic results, such as Sen’s Liberal Paradox and Kaplow and Shavell’s “Anti-Fairness” Theorem, that take on a different significance in light of our results. Section 8 draws attention to one of the most important reasons cycles are problematic—the widespread opportunities they create for manipulation. It also provides some examples of the kind of restructuring economics would need to undertake in order to properly accommodate the law. However, the full development of these points is left for future work. Section 9 extends our result to cases in which a decision maker can be indifferent among several options or faces more than two options. In the case of choices involving multiple options, the cycling condition is replaced by a more general rationality condition, known as WARP, which is the axiomatic counterpart of orders in choice. We illustrate this phenomenon with an example of a WARP violation in law that does not result from a cycle. (In other words, the possibility of choices that involve more than two alternatives adds a further interesting type of seeming irrationality to the making of legal choices). In Section 10, we take up a very general concern someone might have about what qualifies as a genuine cycle. Section 11 concludes. Proofs are in the appendix.

## 2 The Legal Cycling Theorem

We begin by defining an option-stratified legal system. We then show that an option-stratified legal system is the only one that is guaranteed to avoid cycles. By contrast, a system that is not option-stratified is guaranteed to exhibit cycling, and, conversely, a system that exhibits cycling is guaranteed not to be option-stratified. Thus, a legal system is free of

cycles if and only if it is option-stratified. After that, we show why all feasible legal systems are bound not to be option-stratified—and therefore bound to exhibit cycling.

## 2.1 Option-Stratified Legal Systems

Let  $A$  be a finite set of alternatives. An *issue*  $B$  is a subset of  $A$  with two distinct elements. Therefore, if  $x \in A$  and  $y \in A$ ,  $x \neq y$ , then the subset  $B = \{x, y\}$  of  $A$  is an issue. Let  $\mathcal{B}$  be the set of issues. A *legal system* is a mapping  $\mathcal{L}$  that takes an issue  $B$ , as input, and returns, as output, a non-empty subset,  $\mathcal{L}(B)$ , of  $B$ . Thus, a legal system is a mapping  $\mathcal{L} : \mathcal{B} \rightarrow \mathcal{B} \cup A$  such that  $\emptyset \neq \mathcal{L}(B) \subseteq B$ . If there are two available options  $x$  and  $y$  then both of them may be legal or only one of them may be legal. The legal system  $\mathcal{L}$  determines which options are legal:  $\mathcal{L}(\{x, y\})$  are the legal alternatives when the available options are  $x$  and  $y$ . If  $\mathcal{L}(\{x, y\}) = \{x, y\}$  then both  $x$  and  $y$  are legal. If  $\mathcal{L}(\{x, y\}) = \{x\}$  then only  $x$  is legal. In section 9, these definitions, and corresponding results, are extended to choices with more than two options. Let  $\mathcal{R}$  be the set of real numbers.

**Definition 1** *A legal system  $\mathcal{L}$  is option-stratified if there is a utility function  $u : A \rightarrow \mathcal{R}$  such that*

$$\begin{aligned} \mathcal{L}(\{x, y\}) &= \{x, y\} \quad \text{if } u(x) = u(y); \\ \mathcal{L}(\{x, y\}) &= \{x\} \quad \text{if } u(x) > u(y). \end{aligned}$$

In other words, a legal system is option-stratified if there is a function that ranks all theoretically possible alternatives from top to bottom, and if someone, choosing from a feasible subset of all these options, is obliged to choose the highest-ranking one. An option outranked by another is illegal. The highest-ranking choices are not necessarily unique. If two alternatives have the same (top-)rank, then they are both legal.

## 2.2 The Law-Abiding Citizen

A law-abiding citizen is a rational agent who is constrained by the law. Let  $P$  be the preference order (i.e., a complete and transitive binary relation over all alternatives) of a law abiding citizen. It ranks all feasible alternatives from top to bottom and  $x P y$  denotes a preference for  $x$  over  $y$ . We also assume that  $P$  is asymmetric. This rules out indifference between alternatives and, so, rules out “spurious cycles” where, for example, the decision maker is indifferent between three legal alternatives and chooses them in a cycle. In Section 9 we show our results when indifference is allowed.

A *choice function*  $C$  is a mapping that takes an issue  $B$ , as input, and returns, as output, an element  $C(B) \in B$ . Thus, a choice function is a mapping  $C : \mathcal{B} \rightarrow A$  such that

$C(B) \in B$ . The law abiding citizen chooses the best option among the legal ones. Thus, given the law abiding citizen's asymmetric preference order  $P$  and the legal system  $\mathcal{L}$ , the law abiding citizen's choice function  $C$  ( $= C_{P,\mathcal{L}}$ ) is such that

$$C_{P,\mathcal{L}}(B) \in \mathcal{L}(B) \text{ and } C_{P,\mathcal{L}}(B) P y \text{ for every } y \in \mathcal{L}(B), y \neq C_{P,\mathcal{L}}(B).$$

Hence,  $C_{P,\mathcal{L}}(B)$  optimizes  $P$  on  $\mathcal{L}(B)$ . When it is clear that we refer to the choice function of the law abiding citizen (and not an arbitrary choice function), we may drop the subscript  $P,\mathcal{L}$  to ease the notation. So, between  $x$  and  $y$ , the choice of the law abiding citizen is  $x$ , (i.e.,  $C(\{x,y\}) = x$ ) if and only if either  $x$  is the only legal alternative (i.e.,  $\mathcal{L}(\{x,y\}) = \{x\}$ ) or both alternatives are legal (i.e.,  $\mathcal{L}(\{x,y\}) = \{x,y\}$ ) and  $x$  is preferred over  $y$  (i.e.,  $x P y$ ).

The choices of law-abiding citizens are based on two principles. Law-abiding citizens are completely rational and order all options with strict preferences. As mentioned, this avoids spurious cycles arising for uninteresting reasons such as indifferences or cyclical preferences on the part of the citizen. Moreover, law-abiding citizens respect the law and do not choose illegal options. They pick their top-ranked option among those that are feasible and legal. We now turn to the question of whether the choices of a law-abiding citizen can be cyclic.

**Definition 2** A choice function  $C$  is **cyclic** if there exist three distinct alternatives  $x, y$  and  $z$  such that

$$C(\{x,y\}) = x; \quad C(\{y,z\}) = y; \quad C(\{x,z\}) = z.$$

The cycle of length 3 is without loss of generality because if there are cycles of any length then there must also be one of length 3.

**Definition 3** A legal system  $\mathcal{L}$  **induces cycles** if there exists a preference order  $P$  such that the resulting choice function  $C_{P,\mathcal{L}}$  of a law abiding citizen is cyclic.

We speak of a legal system *inducing* cycles to make it clear that we are dealing with perfectly rational decision-makers. If there were no law, as in the special case of a legal system that makes all options legal, there would be no cycles.

## 2.3 Main Result

**Legal Cycling Theorem** Consider the case where there are at least three distinct alternatives. Then,

- (a) No option-stratified legal system induces cycles.

However,

(b) Any legal system that is not option-stratified induces cycles.

**Proof:** See Appendix.

The Legal Cycling Theorem is a full characterization of the legal systems that induce cycles. The upshot of the theorem is this: A legal system needs to tell a citizen which among a set of options he faces is legal. One possible way of picturing his situation is the option-stratified system. Namely, we assume that there is a ranking of all possible options citizens might face and we picture the legal system as requiring a citizen to choose the highest-ranked option among those available to him. If several are ranked equally highly, he gets to choose among them as he pleases, i.e., in accordance with his preferences. It is probably fairly intuitive that this type of system, which so closely resembles the usual choice situation in economics, will not lead to cycling. What seems less intuitive and more interesting is the second part of the Legal Cycling Theorem—that a legal regime that is not susceptible to cycling will necessarily be capable of being reduced to this picture, in other words, that unless it can be thought of as option-stratified, it will necessarily exhibit cycles.

### 3 Cycling in Actual Legal Regimes

Let us now look at examples of cycles as they arise under the common law. We look at four legal doctrines and some cycles they each can generate: duress, self-defense, necessity and negligence. These doctrines are in no way peculiar to the common law. Every legal regime known to us, indeed every legal regime conceivable to us, has these doctrines. They seem to represent basic, culture-insensitive facets of human morality which legal regimes cannot but help reflecting. After presenting these doctrines and the cycles they give rise to, we explore two strategies that might suggest themselves for getting rid of the cycles. The first strategy, consistent with the Legal Cycling Theorem, fails and simply results in producing new and different cycles. The second strategy succeeds but, consistent with the theorem, renders the system option-stratified. This second kind of failure is particularly important to understand because it reveals just why option-stratified systems are not really feasible.

#### 3.1 Cycling and Duress

The defense of duress is available to a defendant who was pressured into committing a crime with the threat of serious pain or injury. If, for instance, someone were threatened with being subjected to serious burns unless he helped out in a bank robbery, he would probably have the duress defense available to him. To be sure, the defense is not available unless the threat is sufficiently serious. Merely being threatened with something that one considers

extremely disadvantageous is not enough. If someone is threatened with the destruction of a piece of property he greatly treasures, even a manuscript he has been working on for many years, that almost surely does not qualify.

The duress defense induces a cycle in the following way. Imagine that the defendant happens to value his manuscript so highly that if a fire were to break out that threatened to consume it, he would not hesitate to rush into the burning building, to salvage it, even at the cost of suffering serious burns. Now we get the following cycle: When choosing between letting the manuscript be destroyed or suffering burn wounds, the defendant will choose to suffer burn wounds. When choosing between suffering burn wounds or participating in the bank robbery, the defendant will choose to participate in the bank robbery, which is permitted by the duress defense. Alas, when choosing between participating in the bank robbery or seeing his treasured manuscript be destroyed by gangsters, he will choose to let his manuscript be destroyed—because that is what the law expects of him under the circumstances, there being no duress defense if he makes the contrary decision. Here then the legal system induces a cycle, that is, it produces intransitive choices in someone who makes rational decisions while subjecting himself to its rules.

The Legal Cycling Theorem implies that the root cause of this cycle is that the doctrine of duress does not allow the legal system to be option-stratified. Now why exactly is that? A key property that any option-stratified system has, but actual legal systems do not, is the following: If when we choose between option  $x$  and option  $y$ , choosing either is legal, and when choosing between  $y$  and  $z$ , either is legal as well, then, when choosing between  $x$  and  $z$ , both must be legal as well. Put formally:

$$\mathcal{L}(\{x, y\}) = \{x, y\} \quad \text{and} \quad \mathcal{L}(\{y, z\}) = \{y, z\} \Rightarrow \mathcal{L}(\{x, z\}) = \{x, z\}. \quad (1)$$

Let's call this property Context Independence, CI. If CI is absent, we have the following:

$$\mathcal{L}(\{x, y\}) = \{x, y\}, \quad \mathcal{L}(\{y, z\}) = \{y, z\} \quad \text{and} \quad \mathcal{L}(\{x, z\}) = \{z\}. \quad (2)$$

Let's call this property Context Dependence, CD. Duress does not satisfy CI: given the choice between manuscript and burns, the legal system permits either. Given the choice between committing robbery and burns, the legal system permits either. But given the choice between manuscript and committing robbery, it only permits one. It is the interaction of this fact with the legal actor's preferences that generates the cycle.

Property (1) must be satisfied by any option-stratified legal systems because for any function  $u$ ,  $u(x) = u(y)$  and  $u(y) = u(z)$  imply  $u(x) = u(z)$ . However, in this example, option  $x$  is to participate in the bank robbery, option  $y$  is to suffer severe burns, and option  $z$  is to lose the manuscript. So,  $\mathcal{L}(\{x, y\}) = \{x, y\}$  because under the doctrine of duress both

the participation in the bank robbery and the enduring of severe burns are legal. In addition,  $\mathcal{L}(\{y, z\}) = \{y, z\}$  because the choice between the burns and the manuscript concerns only the decision maker and both options are legal. Finally,  $\mathcal{L}(\{x, z\}) = \{z\}$  because the duress defense does not apply to participation in the bank robbery if the alternative is to lose a manuscript. That is, property (1) does not hold. Instead,  $(CD)$  holds and the legal system is not option-stratified.

This is worth restating: The legal system here is not option-stratified mainly because the defense of duress is context-dependent. That is, whether or not the defense of duress applies depends not only on what is done and its consequences, but also on the available alternatives. The defense of duress for participating in the bank robbery (option  $x$ ) holds if the alternative is  $y$  (to suffer severe burns), but not if it is  $z$  (to lose the manuscript). When this context-dependence interacts with the decision maker's exercise of his preferences, a cycle results. To summarize, whenever the law satisfies  $(CD)$  and a law abiding citizen ranks  $x$  above  $y$  above  $z$  then, consistently with the Legal Cycling Theorem, the resulting choices produces the cycle

$$C(\{x, y\}) = x, C(\{y, z\}) = y \text{ and } C(\{x, z\}) = z. \quad (\text{C})$$

### 3.2 Cycling and Necessity

The defense of necessity is similar in structure but different in content from the defense of duress. It is available to someone who has a difficult choice to make and chooses to break the law rather than suffer, or inflict, some serious harm that is more serious than the harm that the law he is breaking is seeking to prevent. In other words, if he can do substantially more good than harm by breaking the law, he is permitted to do so. (Note that this is different from the duress defense which applies even when one is doing more harm than one is threatened with.) If, for instance, someone is hiking in the mountains and can only avoid starvation by breaking into a mountain cabin to help himself to its supplies, the defense of necessity would exonerate him. Like the defense of duress, necessity is only available if the injury being prevented by committing the offense is sufficiently serious.

We can generate a cycle because any context-dependent law is prone to cycles and the necessity defense evidently is context-dependent: When choosing between committing a moderately serious crime or suffering the risk of a dire calamity, the defendant is allowed to do either. When choosing between committing a moderately serious crime or suffering some non-calamitous minor set-back, he can only choose the latter. But when choosing between the non-calamitous minor set-back and the risk of a major calamity, he once again can choose either. That violates CI and, hence, guarantees the possibility of a cycle.



What might such a cycle look like? To see how this might happen, let's refine the mountain cabin example somewhat. Suppose that to make this a reasonably safe climb, or rather, to ensure the safety of his descent, which is the harder part, the hiker needs certain type of equipment, which he lacks. His desire to climb, however, is sufficiently great that he chooses to embark on the climb anyway. Somewhat more formally, given the choice between alternative  $z$ , foregoing the climb, and alternative  $y$ , risking death, he chooses the latter. Now suppose that when he reaches the mountain top, he comes across a mountain cabin that happens to contain the equipment necessary for a safe descent. He now faces the choice between what we labeled alternative  $y$ , risking death, and a new alternative  $x$ , breaking into the mountain cabin and helping himself to that equipment. Because he is able to invoke the defense of necessity, he chooses  $x$  over  $y$ . Now finally suppose that he were to find himself confronting the choice between alternative  $x$  and alternative  $z$ , that is, between breaking into the mountain cabin, on the one hand, or foregoing the climb, on the other.<sup>2</sup> He would therefore not be able to claim the necessity defense, and would therefore decline to choose alternative  $x$ , breaking into a cabin to obtain climbing equipment, over alternative  $z$ , foregoing the climb.

The logic underlying the cycle induced by the necessity doctrine is the same as in the cycle induced by the duress doctrine. In both cases, the resulting law is not option-stratified because property (CI) does not hold. Instead, (CD) holds mainly because the doctrine of necessity is also context-dependent. Whether or not the defense of necessity applies depends on what is done, its consequences and also the available alternatives. The defense of necessity for breaking into the cabin (alternative  $x$ ) holds if the alternative is  $y$ , an unsafe descent, but not if it is  $z$ , foregoing the climb.<sup>3</sup>

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<sup>2</sup>One might wonder what sort of scenario would be required to generate this kind of choice situation. Suppose that the cabin containing the equipment needed for a safe descent is located at the foot of the mountain, rather than at the top. This means that if he chooses to break in, he would be doing so when his available alternatives include the option of not climbing the mountain at all.

<sup>3</sup>Now there is something here that might make some readers uneasy about this claimed cycle. Isn't breaking into the cabin at the top of the mountain a different alternative from breaking into the cabin at the foot of the mountain? Why do we treat these as the same alternative? We do so simply as a matter of convenience. We could modify the example, at the cost of making it more artificial, by arranging it such that there is only one cabin, the one at the foot of the mountain, that contains the necessary equipment, but that the equipment could be somehow forcibly removed at a distance, say, by calling a friend, a mountain climber who has the necessary equipment for himself but who would need to break-in the mountain cabin in order to get the additional equipment that can save the first climber.

### 3.3 Cycling and Self-defense

To avoid getting seriously injured from someone’s attack on him, the defendant is allowed to seriously injure him in turn. He is not in general allowed to defend an attack on his property—e.g. his manuscript—by the use of “deadly force,” which refers to force that might seriously injure the attacker (as opposed to actually kill him). Now suppose that he is willing to incur serious injury to protect his manuscript from great harm. Once again we get a cycle. We would observe the defendant when choosing between getting injured or suffering damage to his manuscript, choosing to get injured instead. When choosing between getting injured or injuring his attacker, we would observe the defendant choosing the latter—injuring his attacker (as he is permitted to do by the doctrine of self-defense). When choosing between injuring someone who is about to destroy his manuscript, or permitting him to destroy the manuscript, he would choose the latter, because that is what the law of self-defense requires of him. In short, the doctrine of self-defense induced a cycle.

In this example, the cycle induced by the doctrine of self-defense has the same logical structure as the cycles induced by duress and necessity. Self-defense is also context-dependent. To use deadly force on the attacker ( $x$ ) is legal if the alternative is to incur a serious injury ( $y$ ), but not if it is to have the manuscript damaged ( $z$ ). Moreover, if the options are  $y$  and  $z$  then they are both legal. Thus,  $(CD)$  holds. It follows that the legal system is not option-stratified, and if a law abiding citizen ranks  $x$  above  $y$  above  $z$  then the resulting choices produce the cycle in  $(C)$ .

### 3.4 Cycling and Negligence

The doctrine of negligence imposes liability on those who harm others through negligent actions. Negligence is generally understood to be the unjustifiable imposition of risk. Criteria of justifiability vary. A commonly invoked one is the “Hand formula:” Does the benefit of taking a precaution exceed its cost?

We should expect negligence to generate cycles because it too is not option-stratified, and therefore violates  $CI$ . Given the choice between suffering a very small risk  $x$  or suffering a large risk  $y$ , the legal actor can choose whichever he pleases (assuming no one else is affected). Given the choice between imposing a small risk  $z$  on someone or suffering a large risk  $y$  himself, then, assuming  $y$  is sufficiently large, he can do either. But given the choice between imposing a small risk  $z$  on someone else, or suffering a *very* small risk  $x$  to himself, he can only choose the latter. That means negligence violates  $CI$  and a cycle can be constructed.

Here is a somewhat involved example of such a cycle: Let us imagine an athlete who suffers an accident during a sports event. If he were to continue playing, he runs the risk

of permanent injury. He chooses to continue to play. In other words, between alternative  $z$ , foregoing the game, and  $y$ , risking permanent injury, he chooses  $y$ . Presumably that is a choice he is entitled to make, since he is the only affected party. Now suppose that, after he has chosen  $y$ , and has finished out the game, it turns out that the risk of permanent injury could in fact be averted if he were swiftly brought to an emergency room by an aggressively driven ambulance, but that such an ambulance would be operating at a significant risk to numerous bystanders. We will assume, however, that this is a trade-off which the Hand formula would endorse and which would therefore not be judged negligent. This means that, as between alternative  $y$ , risking permanent injury to the athlete, and alternative  $x$ , imposing a significant risk on numerous bystanders, the athlete would be permitted to opt for  $x$ . Finally, let us imagine a scenario in which he has to choose between  $x$  and  $z$ . How might that happen? Well, let us suppose that the accident he suffers does not pose a risk of permanent injury but simply takes him out of the game, unless he is provided with certain equipment, or treatment, which could only be provided in time by sending a car to the stadium that would have to be driven in the same aggressive manner as the ambulance, posing the same risk to bystanders. Presumably that would not be allowed. In other words, between imposing the self-same risk on bystanders as the ambulance, alternative  $x$ , and foregoing playing in the second half of the game, alternative  $z$ , he is obligated to choose the latter. (Put more simply, imposing the self-same risk on bystanders for the sake of averting permanent injury is permitted, but doing so for the sake of continuing the game is not. However, since he is allowed to not avert the risk of permanent injury rather than forego playing the game, we get a cycle.)

Although the examples are very particular, they are constructed from a very general recipe that can be widely applied, which means that there is nothing rare or unusual about these cycles. The recipe is the following: There are a series of options which a decision-maker cares about to varying degrees—e.g., his manuscript, his physical safety, and not getting involved in a bank robbery. Each of these options has, in common moral and legal parlance, an interest associated with it, that is, in describing the situation we are led to refer to the decision-maker's interest in his manuscript, his interest in his physical safety and the bank's interest in not getting robbed. The relevant legal rules provide a ranking of these interests. They would generally put the bank's interest in not being robbed ahead of the defendant's interest in not having his manuscript destroyed; they would put the defendant's interest in his physical safety ahead of his interest in his manuscript, and they would put the defendant's interest in his physical safety ahead of the bank's interest in not getting robbed. That's of course a perfectly transitive ranking. What induces the cycle is that in choosing between the manuscript and his body, the defendant is allowed to choose what he prefers most, rather than what he has the greater legal interest in. *Anytime we inject the possibility*

*of someone's choosing what he has a lesser interest in, but greater desire for, over what he has a greater interest in, but lesser desire for, a cycle like the above may result.*

Using the conceptual framework of the proof of the legal cycle theorem we can appreciate more clearly what gives rise to cycles. What we call interests correspond to a function that ranks all options. So, if the options are: (a) his manuscript, (b) his physical safety, and (c) not getting involved in a bank robbery, his interests rank  $b$  over  $c$  over  $a$ . However, the law does not require him to always take the highest interest option. In this example, this is only so in the case of the choice between  $a$  and  $c$  (where he is required to choose  $c$ ). The other choices are left to the decision maker. Hence, even if interests are perfectly ranked, legality is not, at least on occasion, determined by the ranking of interests. Sometimes the decision maker is allowed to choose an option with lower interest (e.g.,  $b$  has higher interest than  $a$ , but our law-abiding citizen chooses  $a$  over  $b$ ). Thus, the law is not option-stratified and, thereby induces cycles.

## **4 Why Option-Stratified Systems are Unacceptable: The Nonresponsiveness Problem**

It will prove illuminating to consider some of the strategies people might follow to try to eliminate cycles. One strategy that probably suggests itself arises from a powerful, but as it turns out, false intuition regarding the root cause of these cycles. It might seem for instance that what generates the duress cycle is the law's rigid assumption that physical safety is always more precious than property. It might seem as though the cycle could be made to disappear by simply making the law less rigid, or coarse-grained, by making the availability of the duress defense depend not on the specific injury being threatened but on the amount of disutility associated with the injury. Thus, one might say that because the loss of the treasured manuscript is as serious to this particular defendant as physical injury is to most other people, he gets to invoke duress when it is being threatened. Correspondingly, one might say that because his physical safety is less precious to him than it is to other people, he does not get to invoke the duress defense when that is what is being threatened. So long as the defendant chooses the manuscript over physical safety, it seems as though the cycle has now been made to disappear.

Alas, a closely related cycle can still be constructed. Suppose the defendant has the choice between doing something which puts his manuscript at risk or puts his body at risk. Inasmuch as his manuscript is more precious to him than his body, we would expect him to put his body at risk. However, that does not take into account the effect which the legal rules have on his decision. Inasmuch as he is entitled to protect his manuscript much

more extensively than he is entitled to protect his body—that is, he is entitled to participate in a bank robbery to avoid its destruction—this might well lead him to choose to put his manuscript at risk rather than his body. The cycle has now been recreated. It should be apparent that an analogous argument can be made about each of other cycles laid out. If we tried to modify the doctrines of self-defense, necessity and negligence by reformulating the law in terms of disutility rather than specific objects (like the body or property), a similar reformulation of the cycle is possible. This is just a special case of the familiar phenomenon of someone making himself more vulnerable because that entitles him to certain special benefits.

Let us now see what happens if we try to eliminate cycles through a different approach. More concretely, let us try to turn our cycle-prone legal system into an option-stratified system. As shown by the legal cycling theorem, this is the only strategy that can effectively eliminate cycles. However, it has extremely unattractive implications. In this sense, this is the more important strategy to explore because it helps reveal why option-stratified legal systems are not really acceptable.

One of the chief difficulties presented by an option-stratified system is what we call the “nonresponsiveness problem.” To be an option-stratified legal system, it has to be the case that whenever we allow a decision maker to choose between various alternatives, they have to be “on a par” as far as the legal system is concerned. The legal system ranks all alternatives, and requires the decision maker to choose among the highest ranked available options. He has leeway only if there are several equally ranked options.

Now let us imagine the following. The decision maker faces certain alternatives  $x$ ,  $y$ , and  $z$ . Each of these alternatives carries certain costs and benefits with it—pros and cons, that is, *but only for him*. They have no effect on anyone else. Presumably we would want him to be able to choose between these. We would in general want a system to allow him to choose among alternatives that affect him only. At least in general, maybe not invariably. Paternalism and other considerations might impose some limitations, but we do not require that all choices that produce negligible effects on others to be legal. We only require that some of these choices be legal. Assuming that we want this to be the case, then, in order for the system to be option-stratified (and, thus, cycle-free) these options have to be deemed to be “on a par” as far as the legal system ranking is concerned. So, consider a choice between  $x$  and  $y$  and assume that, if these are the available options, they are both legal. Then, if the law is option-stratified, they must be equally ranked—or the decision maker cannot freely choose among them.

Next let us picture a situation in which a further option  $w$  is injected. This option has significant consequences to others, or rather: choosing  $w$  means sparing that other person certain risks or costs. This is the typical kind of situation contemplated by the negligence

doctrine: either the defendant does what generates certain benefits to him (option  $x$ ) or he does what avoids the risk to others, but deprives him of his benefits (option  $w$ ). Unlike the choice between  $x$  and  $y$ , the choice between  $x$  and  $w$  does have consequences to others. In the latter case, to choose  $x$  means to reject  $w$  and so to let another incur certain risks. Let us suppose that he would be permitted to choose  $x$  over  $w$ . Presumably that would be based on some sort of comparison between the benefits to him of  $x$ , and the risks for the other person if he rejects  $w$ .

Next let us suppose he faces the choice between  $y$  and  $w$ . The  $y$ -option is associated with a different package of costs and benefits for our defendant. Depending on exactly how those costs and benefits compare with those associated with the  $w$ -option (which affect the other person in this set-up), we might or might not want to let the defendant choose  $y$  over  $w$ . And yet, if the system is option-stratified, he *must* treat  $x$ ,  $y$  and  $z$  as equivalents. If we allow him to choose  $x$  over  $w$ , we must also allow him to choose  $y$  over  $w$  and  $z$  over  $w$ , or we give rise to cycles. Hence, if  $x$  is, say, worth a million dollars to the decision maker and  $y$  is, say, worth one dollar and  $z$ , a negative amount then, when the alternative is to spare someone some risks, either he is allowed to do any one of these or none of them. In sum, the negligence doctrine would have to be insensitive, or totally nonresponsive, to use a slightly more technical term, to the degree of benefit an option has for the decision maker when determining whether he is allowed to choose it, which seems bizarre.

Nothing in this hinges on the particular doctrines being considered. The doctrines of necessity, self-defense and duress, if they were to become option-stratified, they would have to be equally nonresponsive to crucial attributes of an option. Virtually all sensible legal doctrines one can think of involve comparing option  $w$  with option  $x$  (if those are the available ones) on some basis or other, to decide which the defendant is entitled to choose; they will involve comparing option  $w$  with option  $y$  (if those are the available ones) to decide which the defendant is entitled to choose among the two; and they will come to different conclusions if  $x$  and  $y$  are sufficiently different. This, however, is precluded if for doctrines to be part of an option-stratified legal system.

## 5 Why Option-Stratified Systems are Impossible, or Nearly So.

In this section, we show the difficulty of combining two or more doctrines to produce an option-stratified system, *even if neither doctrine by itself induces cycling*. The only significant pre-condition of our result is one we call doctrinal unanimity. That is, when all doctrines agree on which options should be legal, the legal system must do what they all agree on,

rather than the opposite. While not restricted to this case, the difficulty we lay bare is a particularly interesting phenomenon when there is no direct inconsistency between the doctrines being combined. Let us suppose they concern themselves with different subjects, and are in full agreement to the extent that they overlap in what they cover. In other words, where one doctrine applies, the other doctrine either does not apply, or if it does, produces the same result. Nevertheless when they are combined, they induce cycles.

Let's start with the observation that an individual doctrine may not be applicable on all issues. For example, doctrines regarding copyright infringements may not be applicable to determine legality on issues regarding the use of deadly force. Henceforth, a *doctrine*  $\mathcal{D}$  is a mapping  $\mathcal{D} : \mathcal{B} \rightarrow \mathcal{B} \cup A \cup \{n/a\}$  such that, for every  $B \in \mathcal{B}$ , if  $\mathcal{D}(B) \neq n/a$  then  $\emptyset \neq \mathcal{D}(B) \subseteq B$ ; moreover,  $\mathcal{D}(B) \neq n/a$  for some issue  $B$ . The expression  $\mathcal{D}(B) = n/a$  refers to the case where the doctrine  $\mathcal{D}$  is non-applicable and so, silent over which options are legal on the issue  $B$ . If  $\mathcal{D}(B) \neq n/a$  then the doctrine is applicable and expresses a viewpoint on the legality of different options when  $B$  are the feasible choices. In this case,  $\mathcal{D}(B)$  are the options that doctrine  $\mathcal{D}$  deems legal.

**Definition 4** A doctrine  $\mathcal{D}$  is a **conditionally option-stratified doctrine** if there exists an utility function  $u : A \rightarrow \mathcal{R}$  such that whenever  $\mathcal{D}(B) \neq n/a$  ,

$$\mathcal{D}(\{x, y\}) = \{x, y\} \text{ if } u(x) = u(y); \tag{E}$$

$$\mathcal{D}(\{x, y\}) = \{x\} \text{ if } u(x) > u(y). \tag{H}$$

Like an option-stratified legal system, a conditionally option-stratified doctrine also ranks all possible alternatives and, whenever the doctrine is applicable, an option outranked by another feasible one is illegal. If two alternatives have the same (top-)rank, then they are both legal, provided that the doctrine is applicable. Legality here refers, naturally, to the viewpoint expressed by the doctrine and not by the final legal system.

We assume that the doctrines we are dealing with in this section are conditionally option-stratified doctrines. This assumption is not necessary for our main result which holds even if we make no assumptions about the doctrines. However, restricting ourselves to conditionally option-stratified doctrines makes the results clearer for the following reason: Let's say that a legal system  $\mathcal{L}$  *adopts* a doctrine  $\mathcal{D}$  if  $\mathcal{L}(B) = \mathcal{D}(B)$  when  $\mathcal{D}(B) \neq n/a$ . Then, a legal system adopts a doctrine when the legal system agrees with the doctrine, when the doctrine is applicable. If a legal system  $\mathcal{L}$  adopts a doctrine  $\mathcal{D}$  that is not a conditionally option-stratified doctrine, then the legal system  $\mathcal{L}$  is not an option-stratified system (and so, induces cycles). This follows because for any utility  $u$ , (E) and (H) cannot hold in the entire domain of issues, if they do not hold in the sub-domain of issues where the doctrine is applicable.

Conversely, if a doctrine  $\mathcal{D}$  is a conditionally option-stratified doctrine, then some option-stratified legal systems can adopt it. The adopting legal system can be directly constructed with the utility function  $u$  (of the conditionally option-stratified doctrine  $\mathcal{D}$ ) and the rules  $(E)$  and  $(H)$ . Thus, if doctrines are restricted to be conditionally option-stratified doctrines, then no single doctrine, by itself, makes the final legal system *not* option-stratified. Thus, no conditionally option-stratified doctrine, by itself, necessarily induces cycles. This restriction makes clear that the difficulty in combining doctrines to construct an option-stratified legal system is above and beyond the difficulty in ensuring that each doctrine, taken by itself, is a conditionally option-stratified doctrine.

Let  $\mathfrak{D}$  be the set of all doctrines and  $\mathfrak{L}$  be the set of all legal systems. An *aggregator*  $\alpha$  is a function

$$\alpha : \mathfrak{D}^n \longrightarrow \mathfrak{L}$$

that maps a profile of doctrines  $(\mathcal{D}_1, \dots, \mathcal{D}_n)$  into a legal system  $\mathcal{L}$ .

**Definition 5** *An aggregator  $\alpha$  maps conditionally option-stratified doctrines into option-stratified legal systems if the legal system  $\mathcal{L} = \alpha(\mathcal{D}_1, \dots, \mathcal{D}_n)$  is an option-stratified legal system whenever the doctrines,  $\mathcal{D}_i, \dots, \mathcal{D}_n$ , are all conditionally option-stratified doctrines.*

The key condition on  $\alpha$  is that it produces option-stratified legal systems. As mentioned, the proviso that this only needs to be so when the doctrines themselves are conditionally option-stratified doctrines makes the results stronger and clearer.

**Definition 6** *An aggregator  $\alpha$  satisfies doctrinal unanimity if for any options  $x$  and  $y$ ,  $\mathcal{L}(B) = \mathcal{D}_k(B)$  whenever these three conditions hold: (i)  $\mathcal{L} = \alpha(\mathcal{D}_1, \dots, \mathcal{D}_n)$ ; (ii)  $\mathcal{D}_k(B) \neq n/a$  for some  $k = 1, \dots, n$ ; and (iii)  $\mathcal{D}_i(B) = \mathcal{D}_j(B)$  for all  $i = 1, \dots, n$  and  $j = 1, \dots, n$  such that  $\mathcal{D}_i(B) \neq n/a$  and  $\mathcal{D}_j(B) \neq n/a$ .*

Thus, an aggregator satisfies doctrinal unanimity if whenever all applicable doctrines agree on what the law should be on an issue then this is the final law on this issue. It may seem natural to assume that  $\mathcal{L}(B) = B$  if  $\mathcal{D}_i(B) \neq n/a$  for all  $i = 1, \dots, n$ . That is, when no doctrine is applicable then all options are legal. However, we do not need this assumption and do not make it.

**Proposition** *Assume that there are three or more options and  $n \geq 2$  (so at least two doctrines must be aggregated into a final legal system). Then, no aggregator satisfies doctrinal unanimity and maps conditional option-stratified doctrines into option-stratified legal systems.*



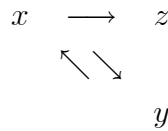
**Proof:** See Appendix.

Under doctrinal unanimity, it is impossible to aggregate more than one doctrine and assure that the final legal system is option-stratified. Hence, the sense in which option-stratified legal system are infeasible is not physical impossibility. Rather, it is that more than one doctrine can potentially be used in the construction of a legal system. As long as there are two or more doctrines, it is not possible to aggregate them and ensure that we end up with an option-stratified system. This follows as long as the aggregation process satisfies doctrinal unanimity. No other conditions are required. The above Proposition regarding aggregation and the Legal Cycling Theorem then yields the ineradicability of cycles, resulting in the Combination Theorem.

**Combination Theorem** Assume that there are at least three options. If multiple doctrines are aggregated under doctrinal unanimity then it is impossible to ensure that the final legal system will not induce cycles.

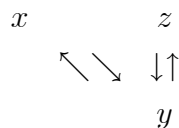
Let us now illustrate the Combination Theorem with some examples. We offer three examples to illustrate this result. The first example is very abstract and schematic. Indeed it can be thought of as a slightly simplified version of our proof. The second is more concrete, but sufficiently generic to indicate that the result should be expected to apply in a wide variety of contexts.

Let us consider three possible alternatives  $x$ ,  $y$ , and  $z$ , as indicated by the vortices in our diagram.



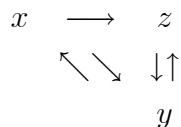
There is a doctrine  $D1$  that ranks  $z$  above  $x$ , in other words, states that given the choice between  $x$  and  $z$ , only  $z$  is legal. The line running from  $x$  to  $z$ , with the arrow pointing towards  $z$ , is meant to indicate that. The doctrine ranks  $x$  and  $y$  equally, meaning that in a choice between  $x$  and  $y$ , it declares both to be legal. The lines running from  $x$  to  $y$ , with arrows pointing towards both  $x$  and  $y$ , is meant to indicate that. There is no line connecting  $y$  and  $z$  because the doctrine does not apply to that choice.  $D1$  could be made into an option-stratified system if we simply made it complete and transitive by drawing such a line between  $y$  and  $z$ , with the arrow pointing towards  $z$ . That possibility is what makes  $D1$  a conditional option-stratified doctrine.

Next let us consider doctrine  $D2$ .



The lines between  $y$  and  $z$ , with arrows pointing towards each, indicate that  $D2$  ranks  $y$  and  $z$  equally. In other words, if those two alternatives were to present themselves,  $D2$  would deem both legal. The lines between  $y$  and  $x$  indicate that  $D2$  ranks them both equally legal. No lines run between  $x$  and  $z$  because the doctrine is inapplicable to that choice. This doctrine too could be rendered completely transitive by extending it, namely by saying that according to  $D2$ , in a choice between  $x$  and  $z$ , both are legal. In other words,  $D2$  is a conditional option-stratified doctrine.

What about combining the two doctrines, consistent with the principle of doctrinal unanimity? That would mean both  $x$  and  $y$  are legal (if both come up together), because the two doctrines agree on that (that is, according to doctrinal unanimity). If  $y$  and  $z$  were to present themselves together, both would be legal, because according to the only applicable doctrine,  $D2$ , that would be true. On the other hand, if  $x$  and  $z$  presented themselves together, only  $z$  would be legal, because according to the only applicable doctrine,  $D1$ , that would be true. Combining  $D1$  and  $D2$  consistent with doctrinal unanimity yields,



That is, property (1) does not hold. Instead,  $(CD)$  holds. But that means that the legal system that emerges from combining  $D1$  and  $D2$  is no longer an option-stratified system and therefore (because of the legal cycling theorem) vulnerable to cycling.

Now let us make up a more concrete example by the simple strategy of filling  $D1$  and  $D2$  with specific doctrinal content. Let us suppose that  $x$ ,  $y$ , and  $z$  are three patients, any two of which might conceivably present themselves simultaneously for treatment in an emergency room, requiring the doctor in attendance to make a triage judgment as to whom to treat first. We will assume that this decision about priority of treatment really matters to the outcome. Let us assume, moreover, that their injuries are of roughly equal severity. Let us also assume that two of them,  $x$  and  $z$ , were both involved in a boating accident,  $x$  being the officer on that boat, and  $z$  a mere passenger. Finally, let us assume that  $y$  is also a ship officer, though not on the boat involved in this accident.

We could imagine there to be two choice doctrines relevant to this situation.

$D$ -Special Duty ( $D - sp$  for short) provides as follows: “As between patients, where one of them owes a special duty to the other (as captains do to passengers, and doctors to patients, and so forth), the one who is owed the duty generally gets priority. Where both belong to the same professional class (e.g. both are doctors, or both are ship officers), priority is to be given according to needs and likelihood of benefitting from treatment.”

$D$ -triage ( $D - t$  for short) provides as follows: “As between patients between whom no

special relationship exists, or patients belonging to the same professional class (e.g. both are doctors, or both are ship officers), priority is to be given according to needs and likelihood of benefitting from treatment.”

Note that  $D - sp$  and  $D - t$  overlap a bit, as legal doctrines often do, although the area of overlap does not seem problematic because they provide for the same thing with regard to the contingency where they overlap (the case where both patients belong to the same professional class). Now let’s consider each doctrine a bit more closely.

Let’s take a closer look at  $D - sp$ .  $D - sp$  would require that, as between  $x$  and  $z$ ,  $z$  be treated ahead of  $x$ , since  $z$  is a passenger and  $x$  is an officer on the ship on which  $z$  was injured, and therefore owes him a special duty.  $D - sp$  would require that, as between  $x$  and  $y$ , either could be treated first, since they are both ship officers. As for the choice between  $y$  and  $z$ ,  $D - sp$  does not apply because there is no special duty and they do not belong to the same professional class.

$D - sp$  is a conditionally option-stratified doctrine because we could make it an option-stratified system simply by requiring that in a choice between  $y$  and  $z$ ,  $z$  should be the only legal alternative.

Let’s now take a closer look at  $D - t$ .  $D - t$  would find that as between  $x$  and  $y$ , both being ship officers, needs and likelihood of benefit should decide and since these are equal, choosing either  $x$  or  $y$  would be legal. As between  $y$  and  $z$ , there being no special relationship between them, and they not belonging to the same professional class, needs and likelihood of benefit will decide. Since those are equal, giving priority to either  $y$  or  $z$  would be legal. As between  $x$  and  $z$ , the doctrine would simply not apply, since there is a special relationship between them.

$D - t$  is also a conditionally option-stratified doctrine because we could make it an option-stratified system by simply requiring that in a choice between  $x$  and  $z$ , both should be legal.

What happens if we combine both doctrines? In the choice between  $x$  and  $y$ , the two doctrines agree that both should be legal, and therefore they both would be. In the choice between  $y$  and  $z$ , the only applicable doctrine,  $D - t$ , declares both options to be legal, and therefore they both would be. Alas, in the choice between  $x$  and  $z$ , the only applicable doctrine,  $D - sp$ , declares only  $z$  to be legal. This means the combination  $D - sp / D - t$  legal regime is not an option-stratified system and therefore, according to the legal cycling theorem, vulnerable to cycles.

Our third example is meant to illustrate that the impossibility of combining doctrines to form an option-stratified legal system holds regardless of whether, in each choice, only one doctrine is applicable or both doctrines are applicable and one doctrine overrules another. Consider another case of triage in an emergency room. Once again there are three injured

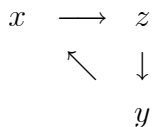
parties,  $x$ ,  $y$ , and  $z$ . Let's say that the injuries are sufficiently similar so that if the doctrine of negligence governs who is to receive priority from the doctor on duty, then he would be free to choose either  $x$ ,  $y$ , or  $z$ . All three options are legal. Now let's say that  $x$  and  $z$  happen to be husband and wife. The husband ( $x$ ) dotes on his wife ( $z$ ) and wants her to be treated ahead of him. Let's say that freedom of contract overrules negligence in the choice between treating the husband or the wife first: the wife must be treated ahead of the husband. However, in the choices between  $y$  and  $z$ , and between  $x$  and  $y$ , there is no contract among the parties and therefore negligence doctrine prevails, which means that as between them, the doctor is free to choose to treat either. This means that once again  $(CD)$  holds and the legal system is not an option-stratified system.

## 5.1 Cyclical Law

There are two ways in which the law can induce cycles. One of them is when  $(CD)$  holds. That is, both options are legal in the choice between  $x$  and  $y$  and in the choice between  $y$  and  $z$ , but only one option is legal in the choice between  $x$  and  $z$ . This is the type of law-induced cycle that we have focused on up to now. It follows from an interaction between the law and the preferences of the decision maker. The other type of law-induced cycle is more direct and occurs when the law itself is cyclical. That is, when

$$\mathcal{L}(\{x, y\}) = \{x\}, \mathcal{L}(\{y, z\}) = \{y\} \text{ and } \mathcal{L}(\{x, z\}) = \{z\}. \quad (CL)$$

In terms of our diagram, this occurs when



In this case, the choices of a law-abiding citizen are cyclical, regardless of the decision maker's preferences. A law-abiding citizen must follow the law. Thus, if the law is cyclical then the choices of a law-abiding citizen must also be cyclical. Consider our proposition regarding aggregation of conditionally option-stratified doctrines. In the proof of this proposition (see appendix) we show that, under unanimity, combined doctrines fail to produce an option-stratified legal system because *both*  $(CD)$  and  $(CL)$  can occur. Thus, when doctrines are combined, the law can become cyclical in some cases, whereas in others cycles result from the interaction of the law and the decision maker's preferences, as in  $(CD)$ . We now illustrate the case where combined doctrines make the legal system itself cyclical, with suitable variations on the examples above.

Let's return to the triage in the emergency room, but now let's assume that  $x$ 's injuries are far more serious than  $y$ 's injuries which are, in turn, far more serious than  $z$ 's injuries.

The negligence doctrine by itself constitutes an option-stratified system devoid of cycling problems. Party  $x$  must be treated ahead of  $y$  which must be treated ahead of  $z$ . If, however, as in our original example,  $x$  and  $z$  are husband and wife who agree that  $z$  must be treated ahead of  $x$  then, by freedom of contract,  $z$  must be treated ahead of  $x$ . Hence, if the law is to follow negligence in the choices involving person  $y$  (where no contract exists), and the principle of freedom of contract (where a legal contract does exist) then we get the cyclical law where  $x$  must be treated ahead of  $y$  who must be treated ahead of  $z$  who must be treated ahead of  $x$ .

This example is actually far more general than it might appear at first. One way of appreciating its generality is to replace the freedom of contract doctrine with another doctrine that formally accomplishes the same thing. For instance, let us change the facts a little. Let us no longer assume that  $x$  and  $z$  are husband of wife. Instead, let us assume that  $x$  attacked  $z$  and the injuries each got are the result of that fight. We might now plausibly adopt an “equitable consideration” doctrine that applies comparatively between  $x$  and  $z$  and gives  $z$  priority over  $x$ . If we assume, as we plausibly might, that the relative priority decreed by the negligence doctrine between  $x$  and  $y$  and between  $y$  and  $z$  remains untouched, we get the same cycle. Negligence ranks  $y$  head of  $z$  and  $x$  ahead of  $y$ , and “equitable consideration” does not come into play with regard to either pair. It does come into consideration, and displaces, negligence as between  $x$  and  $z$ , thus producing a cycle.

A more commonplace doctrine to take the place of either freedom of contract or the “equitable consideration” of the previous paragraph would be a fiduciary duty doctrine (such as might prevail between a captain and his passenger on a ship, or between most professionals and their clients), which typically prohibits a party from benefiting at the expense of another, even if that is cost-justified, in the sense that his benefits would exceed the other party’s loss. If we posit that kind of fiduciary relationship between  $x$  and  $z$ , then it would operate in the same way to produce a cycle. In other words, negligence allows  $y$  to prevail over  $z$  and would allow  $x$  to prevail over  $y$ . The fiduciary doctrine, however, would allow  $z$  to prevail over  $x$ . If  $x$  is the decision maker, we now have a fairly typical risk-creation scenario in which someone, namely  $x$ , has to make a decision that will affect other parties and has to choose one among several feasible way of distributing risks among them. If such a scenario is subject to the negligence doctrine and some other doctrine that operates like freedom of contract, “equitable consideration,” or fiduciary duty doctrine, we get cycles of just the sort the theorem contemplates.

These examples are actually all structurally similar to a cycling problem long familiar to the law, but mistakenly thought to be somewhat exotic, namely the problem of circular priorities that can arise in property law and in the law of secured transactions. Owner  $O$  first sells his property to Buyer 1; next he fraudulently sells the same property to Buyer 2,

and finally he sells it a third time to Buyer 3. In the end, the authorities have to decide who among the buyers has priority over whom. They will do so by resorting to several appealing doctrines, each of which taken by itself may be an option-stratified doctrine, but which in combination no longer are. The first doctrine provides that prior purchases prevail over subsequent purchases. A second doctrine provides that if a purchaser files a record of his purchase in an official “record-book,” he prevails over one who did not, and that if both purchasers filed, the first to file prevails. Finally, a third doctrine provides that if a later purchaser files his purchase in the record book, ahead of previous purchaser who did not, but in fact has notice of the prior purchase, he loses to the prior purchaser. A cycle arises in the case in which Buyer 1 buys, but does not file, Buyer 2 files but knows that Buyer 1 bought, and Buyer 3 buys, files, but does not know about any of the prior buyers.

## **6 Relationship of All This to the Social Choice Literature**

It is only natural to wonder how exactly our theorem relates to social choice theory, especially Arrow’s theorem, to which it has some connection. We will here try to spell out some of the connections and differences.

Both Arrow, social choice theory, and our results deal with intransitivity and how it can arise. That is probably the most marked and notable area of overlap. But there is also a notable difference in the particular way in which intransitivity gets into the picture, as well as of course the subject matter to which it pertains, collective decision-making in social choice theory, legal systems in this paper.

One rough-and-ready way of describing Arrow’s theorem is to say that it shows us that if one tries to aggregate the preference orderings of different people, one often gets results that are either intransitive, or dependent on irrelevant alternatives, or peculiar in various other ways. Many of the important impossibility results proved in the wake of Arrow’s have the same general character. Although our results too are concerned with intransitivity, our first main result, the Legal Cycling Theorem, does so in a different way than the Arrovian literature. Whereas that literature starts out with a series of orders as the “input,” as it were, and shows that the “output” is intransitive, in contrast, we start with something very general, a legal system on which we basically impose no restrictions (e.g., no requirement that it be an order), and then show that if one were to impose a single, mild-seeming requirement on it, the prohibition on cycling, one ends up with exactly one very peculiar type of legal system, what we call an option-stratified system.

Our second main result, the Combination Theorem, is more directly in line with the

Arrowian approach because it shows how combining various orderly rankings, those imposed by a variety of transitive doctrines, can result in an intransitive ranking. Here too, however, there are important contrasts, because we dispense with some of the standard assumptions relied on to varying degrees in that literature, such as independence of irrelevant alternatives, liberalism and so on. However, we do make use of an assumption resembling the Pareto principle in our Combination Theorem, namely the assumption that when all pertinent doctrines agree with respect to an outcome, that should also be the outcome decreed by the legal system (Doctrinal Unanimity).

It should also prove helpful to relate our result to the specifically legal literature that has been inspired by social choice theory. Arrow's theorem initially attracted legal scholars' attention because it was about collective decision-making, and law is the product of collective decision-making by legislatures and by multi-member courts. Since Arrow proved that rational peoples' preferences could not be readily aggregated into something resembling a single person with conventionally rational preferences, it was recognized that this rendered problematic the laws' tendency to treat the pronouncements of legislatures and courts as though they reflected the coherent intentions of a single person. (Spitzer 1979; Easterbrook 1982; Kornhauser and Sager 1986).

But this was not the only way in which Arrow-type results were seen to be relevant to law. In the second use legal scholars found for social choice, they followed the lead of economists and decision theorists who understood that Arrow's insights had implications not only for collective decision making but for individual decision-making, inasmuch as it involved the aggregation of multiple criteria that functioned somewhat analogously to the preferences of individual voters. Several scholars, realizing that legal decision-making is a kind of multi-criterial decision making, then started to wonder about the implications Arrow-type results might have for law, exploring for instance the difficulties judges and administrators will run into when they try to devise rules meeting a combination of different desiderata, as they try to aggregate the different underlying goals, values, and principles behind these rules in acceptable ways. (Spitzer 1979; Chapman 2003; Katz 2011; Miller and Rachmilevitch 2014) Bruce Chapman, in particular, speculated that potential intransitivities lurk behind many legal phenomena and should not be viewed as a blemish but a crucial feature that needed to be explored further. (In a related vein, though by a rather different line of argument than ours, the philosopher Larry Temkin has suggested that all moral reasoning routinely violates transitivity (Temkin 2011).

We too are looking at legal decision-making as a type of multi-criterial decision-making, although somewhat differently from the way it has been done before. We don't start out with any "goals", "principles" or "values" that the legal doctrines in question seek to aggregate. Instead we simply take as given whatever set of doctrines the legal system happens to

contain. We then impose a simple requirement on the decision-making of people subject to this system, namely that it not violate transitivity or ordered choice (i.e., WARP), and we then show that only a very unrealistic, unattractive legal system such as has never existed, the option-stratified one, will meet this requirement.

There is an additional connection between this paper and social choice. So far, the preference  $P$  is taken to be the one of an individual: the law-abiding citizen. However, nothing prevents this preference  $P$  to be the one of a group or the entire society. Social choice theory often shows that social preferences cannot always be ordered. The results in this paper show an additional difficulty. Consider the case that social preferences can be expressed by an order (e.g., a social welfare function). Now assume that society's final choice is the one that maximizes social welfare function among feasible and legal options. Even if, by assumption, social preferences are ordered, social choices can be cyclic (or more generally, violate WARP and, hence, are non-ordered) if the legal system is not option-stratified. In this sense, the difficulty imposed by legal constraints is above and beyond the traditional difficulty in social choice.

## 7 Classic Results Revisited

As we have shown, cycles are not necessarily the result of irrationality, but may simply follow from the fact that plausible legal regimes are not option-stratified. This new perspective on cycles makes it natural to revisit fundamental results obtained when cycles are ruled out. Here we focus on Sen's liberal paradox and Kaplow and Shavell's "anti-fairness" argument (Sen 1979; Kaplow and Shavell 2002).

### (1) Sen's liberal paradox.

Sen's so-called liberal paradox reveals a conflict between a legal system's granting its citizens rights of an even rudimentary nature and respecting the Pareto principle. If the system grants rights and if it respects the Pareto principle, it gives rise to a cycle. Sen's claim has often been attacked on the ground that he had an implausible, eccentric conception of rights. While we don't believe that criticism valid, there is no reason for us to address it here. Instead we note that our result casts the implications of Sen's result in a rather different light. His result suggested that, under transitivity, we must choose between rights and the Pareto principle. Now, a commitment to both rights and the Pareto principle can lead to cycles just as the combination of two or more doctrines can produce non-option-stratified systems and non-option-stratified systems can induce cycles. However inasmuch as all plausible legal regimes produce such cycles, and live with them, we can say the same thing about Sen's situation. We may be able to have both elementary rights and the Pareto



principle despite the fact that they lead to a cycle. The existence of a cycle in and of itself does not seem a compelling reason to rule either of them out of bounds, or to force a choice between them.

(2) Kaplow and Shavell on fairness versus welfare.

Kaplow and Shavell showed that if one combines the Pareto principle with any kind of fairness-based principles that will produce a cycle. They therefore argue that fairness-based legal principles should be rejected. However, in our model, if any two doctrines are combined that can lead to a cycle. So, if the Pareto principle is combined with just about any doctrine, that can produce a cycle. Thus our result casts a different light on that implication in much the same way that it does with respect to Sen. We can say about Kaplow and Shavell’s result what we say about Sen’s result, simply replacing “rights” with “fairness-based principles”: Inasmuch as all plausible legal regimes produce cycles, and live with them, the existence of a cycle in and of itself does not seem a compelling reason to rule the choices that give rise to them out-of-bounds. That is not to say that the intransitivity is never a problem. It is just that transitivity cannot be taken for granted. There would seem to be reasonable and unreasonable forms of intransitivity and the challenging question in each case in which the specter of intransitivity rears its head is to find out which it is. That is something that remains to be explored in connection both with Sen and with Kaplow and Shavell.

## 8 Legal Strategizing and the Concern with Cycles

Our argument that cycles are inevitable does not however mean that there are no legitimate concerns with cycles. It is generally understood that where there is cycling there are ample opportunities for strategic behavior—for manipulation. This is most familiar in the voting context, in which control of the agenda, and especially the sequence in which certain issues are voted on, can greatly influence the outcome. Cycling in the legal context would seem to harbor those same possibilities. It is part of our research agenda to explore the various strategic opportunities produced by cycling in law. In a future paper, we will show that, just like in the voting context, cycles induced by law give law-abiding citizens opportunities to game the law. Hence, as a corollary of the results in this paper, the opportunities to manipulate the law are also ineradicable.

The analysis of legal strategizing produced by cycles is beyond the scope of this paper. So, we will simply draw the reader’s attention to the most immediately obvious one: By manipulating the order in which certain choices are made a great deal of what might look like circumvention of rules is made unavoidable. We know that in any cycle, it should be possible to end up where you want to end up, regardless of where you start out, so long as you

make the right sequence of choices. Let us illustrate that with an artificial, but nonetheless illuminating example using the duress situation described above.

The original version of the duress example involved a defendant who is threatened with something very painful unless he helps the people who have made the threat commit some crime, for which he can validly claim the defense of duress. By contrast, we noted, if he had been threatened with the destruction of a treasured manuscript he has labored over for many years, and if, to avert the manuscript's destruction, he had assisted them in their planned crime, he would not qualify for the defense. This gave rise to a cycle because the defendant is allowed to choose to endure great pain in exchange for protecting his manuscript, he is allowed to commit a serious crime so as to avoid being subjected to the painful treatment, but he is not allowed to commit the crime so as to prevent his manuscript from being destroyed. Here is how he might exploit this intransitivity strategically: Suppose the defendant is determined to do the equivalent of saving his manuscript by committing a crime. What he does is to pay off the people who are seeking to recruit him for a crime with money that he borrows from a loan shark. This loan shark in turn demands that he commit a serious crime as a way of extinguishing his debt, which he cannot pay. But if he committed a crime to escape the loan shark's threats, he would most likely qualify for the defense of duress, because at this point he is doing so not to protect his manuscript but to avert great physical harm.

The example is of course contrived. But the contrivance is the sort that is bound to have more realistic counterparts. Wherever there is intransitivity, there is an opportunity, at least if the context is even mildly propitious, for strategic exploitation of this sort. The full scope of such opportunities, including the exploitation of "menu" effects and related phenomena, we plan to explore in another paper.

We now mention two more topics for future research just in passing. The first is legal reasoning. Legal reasoning often depends on pointing out that if one commits to a series of principles they might clash, in the sense that they lead to cycles, and one of them should therefore be abandoned. Presumably some of these types arguments still pass muster, but others may now have to be rejected. It is a task for the future to figure out which arguments from intransitivity prove that a legal rule must be rejected and which do not. A second topic is that of rights. People are often very skeptical of the concept of rights ("nonsense of stilts" in Bentham's famous phrase) because it leads to various kinds of intransitivities. What is known as the deontological paradox is perhaps the clearest example. These objections need to be rethought. Perhaps many of the objectionably paradoxical features of rights are simply paradoxical features that all legal systems that are not option-stratified exhibit.

## 8.1 Some Implications for Economics

Transitivity is a central tenet in economics. Thus, basic economic principles must be revisited if legal restrictions are to be taken into account. Consider for example, revealed preference theory. An elementary idea is that if  $x$  is chosen over  $y$  and  $y$  is chosen over  $z$  then  $x$  must be preferred to  $z$ . This holds if there are no legal restrictions, but may not hold otherwise. It is known exactly how to infer preference from choice when the law is not taken into consideration, but not when there are legal restrictions. The full characterization of how to infer the preferences of the law-abiding citizen from his choices is motivated by the results of this paper, but is left for future work.

This special case of revealed preference theory is, however, just an example of the type of restructuring economics would need to undertake in order to properly accommodate legal restrictions. In this paper, we limited the analysis to decision under certainty, in which case the decision maker is typically assumed to satisfy WARP (and, hence, make transitive choices). Under uncertainty, the decision maker in economics is typically assumed to satisfy additional axioms (the axiomatic structure of Savage and of von Neumann-Morgenstern being the most prominent examples). These are building blocks on which economics rests. But just as transitivity may no longer hold when the law is taken into account, other basic principles (e.g., monotonicity) may also not hold under legal constraints. Hence, the axiomatic structure of the law-abiding citizen both under certainty and under uncertainty may be quite distinct from the traditional axiomatic structures that ignore the law. The development of these new axiomatic foundations that would fully integrate law and economics is also beyond the scope of this paper, but can be motivated by our results.

## 9 Extensions

### 9.1 Choices with more than two options

In this section, we extend our main results to choices with two or more options. So, in this section, let an issue  $B$  now be a subset of  $A$  with two or more elements, all of which are different from each other. Unless otherwise stated, other definitions from previous sections remain valid. Thus, a legal system is still a mapping  $\mathcal{L} : \mathcal{B} \rightarrow \mathcal{B} \cup A$  such that  $\emptyset \neq \mathcal{L}(B) \subseteq B$ . However, to differentiate the case of binary choice from the general case we refer to  $\mathcal{L}$  as a full legal system if choices are not necessarily binary. The definition of what it means for a full legal system to be option-stratified is a direct extension of our previous definition.

**Definition 7** *A full legal system  $\mathcal{L}$  is **option-stratified** if there is an utility function  $u :$*

$A \longrightarrow \mathcal{R}$  such that for every issue  $B$ ,

$$x \in \mathcal{L}(B) \Leftrightarrow u(x) \geq u(y) \text{ for every } y \in B.$$

As before, a full legal system is option-stratified if there is a function that ranks all theoretically possible alternatives and deems legal the top ones and only the top ones.

If choices may involve more than two options there are different types of behavior that are inconsistent with the choices of a rational agent subject to physical constraints. This type of behavior is known as WARP (the Weak Axiom of Revealed Preference) violations. Formally

**Definition 8** *A choice function  $C$  **violates WARP** if there are two issues  $B$  and  $B^*$  such that*

$$B \subseteq B^*, C(B^*) \in B \text{ and } C(B) \neq C(B^*).$$

A violation of WARP occurs when the choice  $C(B^*)$  in the super-set  $B^*$  is in the sub-set  $B$ , but it is not chosen. A cycle implies a WARP violation because if  $C(\{x, y\}) = x$ ,  $C(\{y, z\}) = y$ , and  $C(\{x, z\}) = z$  then no matter which choice is made on the issue  $\{x, y, z\}$  there is a WARP violation. A choice function such as

$$C(\{x, y\}) = x, C(\{y, z\}) = y, C(\{x, z\}) = x, C(\{x, y, z\}) = y.$$

is not necessarily cyclical, but violates WARP because  $y$  is rejected against  $x$  in the binary choice, but  $y$  is chosen over  $x$  in the choice between  $x, y$  and  $z$ . Whenever the choices violate WARP they cannot be ordered because  $x$  is chosen over  $y$  in some issue and  $y$  is chosen over  $x$  in another issue.

**Definition 9** *A full legal system  $\mathcal{L}$  **induces WARP violations** if there exists a preference  $P$  such that the resulting choice function of a law abiding citizen  $C_{P, \mathcal{L}}$  violates WARP.*

We also speak of a legal system inducing WARP violations because if there were no law, there would be no violations of WARP (Samuelson 1938).

**Extended Legal Cycling Theorem** Consider the case where there are at least three distinct alternatives. Then,

- (a) No full legal system that is option-stratified induces WARP violations.

However,

- (b) Any full legal system that is not option-stratified induces WARP violations.

**Proof:** See Appendix.

The Extended Legal Cycling Theorem characterizes all legal systems that induce WARP violations. It is the counterpart of the Legal Cycling Theorem when choices involve two or more alternatives. The Combination Theorem is an impossibility result that holds in the case of binary choice and, therefore, it also holds in the case of choices involving two or more options as well.

What exactly this theorem means, and what it adds to the original Legal Cycling Theorem will become clearer with the help of an (admittedly artificial) example. Let us return to a situation of triage in which we have three patients. Al, Bea, and Chloe are competing for some scarce medical resource. It could be the ER doctor's attention, or a transplant organ, or some medical equipment. Perhaps the most realistic version of what we have in mind would be the Seattle God Committee, which in the early days of dialysis had to decide which patients would have access to scarce dialysis slots. Let us suppose that the decision-maker uses a rule that works as follows: All patients are rated on three scales—the severity of their injury, the benefit they are likely to derive from treatment, and miscellaneous equitable factors that argue in their favor. All patients under consideration for the one and only treatment opportunity during a particular period are then ranked on each of these relevant factors, and the patient who has the largest number of factors cutting in his favor gets treated during that particular period. If it is a tie, the doctor gets to decide according to his own preferences, or by lot.

Now let us suppose that if we were to rank Al, Bea, and Chloe on each of these factors, this is what we get:

Severity: Bea, Al, Chloe

Benefit: Chloe, Al, Bea

Equity: Al, Bea, Chloe

If the only choices the decision-maker might end up facing are binary ones, because it only ever happens that two patients present themselves at one time, the rule we have translates into something resembling a majority voting system: whoever has the support of two or more factors, prevails. The only relevant theorem would then be our first Legal Cycling Theorem, which tells us that unless this system is option-stratified, it will be prone to cycles. That is indeed the case, since we know that majority voting, which this is equivalent to, can result in cycling. In this case, however, no such cycle results because of the particular way in which the three patients happen to be ranked. If Al and Bea were to have their claims to treatment evaluated, Al would win (there being two factors in his favor). If Bea and Chloe were to have their claims compared, Bea would win (there being two factors in her favor). If Al and Chloe were to be compared, Al would win, and thus everything here is nicely transitive. No real surprise there: cycles only occur under certain circumstances. Can we therefore breathe

a sigh of relief and conclude that at least when the three patients are ranked as they are, the legal system will not produce any odd results? The Extended Legal Cycling Theorem tells us that there is still something to worry about. We can see what it is, if we consider the possibility that all three patients will present themselves for treatment simultaneously. Now we have a tie, since each patient has exactly one factor on which he ranks first. The significance of that fact is that the rule we have in place, although it is not generating a cycle in this case, does violate WARP.

Formally, we have the following: Let  $x$  be treat Al first,  $y$  be treat Bea first and  $z$  be treat Chloe first. Then, in this example,

$$\mathcal{L}(x, y) = x, \mathcal{L}(y, z) = y, \mathcal{L}(x, z) = x, \mathcal{L}(x, y, z) = (x, y, z).$$

So, if the doctor prefers to treat Bea first (perhaps due to a preference for severity over benefit and equity), e.g., if, say  $y P z P x$  and the doctor is law-abiding then the doctors' choices are

$$C(\{x, y\}) = x, C(\{y, z\}) = y, C(\{x, z\}) = x, C(\{x, y, z\}) = y$$

and, hence, violate WARP, but are not cyclic. In the same way, if the doctor prefers to treat Chloe first, e.g., if, say,  $z P y P x$  and the doctor is law-abiding then the doctors' choices are non-cyclic but violate WARP, albeit in a different way. The choices now are

$$C(\{x, y\}) = x, C(\{y, z\}) = y, C(\{x, z\}) = x, C(\{x, y, z\}) = z$$

and so  $z$  is chosen over  $x$  and  $y$  although  $z$  is rejected against  $x$  and  $z$  is rejected against  $y$ .

This is a disturbing possibility, since it opens up opportunities for extensive manipulation just as cycling does. Someone could try to influence the choice simply by temporarily injecting a third alternative into the choice set. It seems a strange and disturbing property of legal systems. In the first example one could reverse the choice between  $x$  and  $y$  but adding  $z$  as an option. In the second example, one could reverse the choice between  $x$  and  $z$  by adding  $y$  as an option. Any violation of WARP, whether it is a cycle or not, makes the choices non-ordered and, hence, vulnerable to manipulation.

How could we prevent this from happening? Only by turning the legal system into one that is option-stratified (and therefore faces all the now familiar difficulties of option-stratified systems). That is what the Extended Cycling Theorem tells us.

Put differently, in legal systems that allow not merely binary choices (which all known systems of course do), there looms an additional possibility that is almost as disturbing as cycling, the violation of the WARP condition. If one wants to keep this disturbing feature out, along with cycling, the only way to do it is to switch to an option-stratified system.

Now the doctrine we have used to make our point is admittedly a bit artificial. It is not traditional law, although not unknown to the law either, at least in such triage settings. An interesting question is to what extent more traditional rules do in fact exhibit WARP violations that are not cycles. We are not sure. We have had some trouble finding them, which is not to say they don't exist. It is easy to overlook entire domains in which a certain odd phenomenon in fact predominates. The one example we have come across is going to be slightly controversial. On one interpretation, the act/omission (or killing/letting die) distinction qualifies as an instance of a WARP violation. In a well known, if controversial, article the philosopher Jonathan Bennet asks why in a case of “a woman in labor who will die unless an operation is performed in which the head of her unborn child is crushed or dissected, while if it is not performed, the child can be delivered, alive, by post mortem caesarean section,” we describe the crushing of the skull as an act but not the actions of the doctor who is writing up his lab report instead of rescuing the mother. Bennet thinks the reason is that the doctor who is writing up his labnotes is taking one of many possible actions that would result in the mother's death, whereas the doctor who crushes the baby's skull is taking the only action that would result in the baby's death. The difference between a death-producing omission and a death-producing act, as he sees it, is that in the former the actor engages in one of many death-causing actions whereas in the latter he engages in one of a very few death-causing ones.

Perhaps a more intuitive example, in support of Bennet's suggestion might be the following scenario: Someone is standing on a platform. To begin with, let us suppose that the platform consists of two “spots” on which he could stand. If he stands on spot  $D$ , the victim will die. If he stands on spot  $L$ , the victim will live. If he chooses to stand on  $D$ , we would describe that as a killing. But suppose that the platform has many more spots, and all of them except one would result in the victim's death. Then we might well be inclined to describe his failure to stand on the one and only  $L$ -spot as a mere omission.

If Bennet is right, that would mean that if we have a choice set  $(D, L)$  in which doing  $D$  would result in the victim's death and  $L$  would result in the victim's continuing to live, then  $D$  is to be considered an act of killing, but if the set contained  $(D, L, D^1, D^2, D^3, D^4, \dots, \text{etc})$  then  $D$  would be considered a letting die, which is a very clear instance of a WARP violation arising under a basic legal doctrine. Formally, let  $x$  be to stand on  $L$ ,  $y$  to stand on  $D$ , and  $z$  be to stand on one of the many places all of which leads to the victim's death. Then, in one possible interpretation,

$$\mathcal{L}(x, y) = x, \mathcal{L}(y, z) = (y, z), \mathcal{L}(x, z) = (x, z), \mathcal{L}(x, y, z) = (x, y, z).$$

Thus, if the law-abiding citizen preferences are  $y$  over  $x$  over  $z$  then we have the (non-cyclic) WARP violation

$$C(x, y) = x, C(y, z) = y, C(x, z) = x, C(x, y, z) = y.$$

One more point seems worth making about our initial triage example, which relates not to the Extended Cycling Theorem, but our earlier discussion of the connection between our results and the social choice literature. The triage example helps illustrate our suggestion that there are various ways in which one can think of the application of legal doctrines as a form of multi-criterial decision-making in which the different criteria that go into making the decision can be viewed as analogous to voting in social choice theory, and one can therefore expect many of the same oddities that beset voting to beset legal systems.

## 9.2 Allowing Indifference

So far, we have not allowed the law abiding citizen to be indifferent between options. In this section, we show a counterpart of the Legal Cycling theorem that holds even if indifference is allowed. As before we make changes in some, but not all, definitions.

In this section, a preference  $P$  is a complete, transitive binary relation (i.e., an order). So,  $P$  may or may not be asymmetric. In principle, there may exist two distinct alternatives  $x$  and  $y$ ,  $x \neq y$ , such that

$$x P y \quad \text{and} \quad y P x.$$

This is the case of indifference between  $x$  and  $y$ . Indifference makes it possible for more than one option be optimal and, therefore, for more than one option to be selected. A *choice correspondence*  $C$  is a mapping  $C : \mathcal{B} \rightarrow \mathcal{B} \cup A$  such that  $C(B) \subseteq B$ . The law abiding citizen choice correspondence  $C$  ( $= C_{P, \mathcal{L}}$ ) is such that

$$C_{P, \mathcal{L}}(B) \subseteq \mathcal{L}(B) \quad \text{and} \quad C_{P, \mathcal{L}}(B) P y \quad \text{for every} \quad y \in \mathcal{L}(B), \quad y \neq C_{P, \mathcal{L}}(B).$$

Hence, as before,  $C_{P, \mathcal{L}}(B)$  optimizes  $P$  on  $\mathcal{L}(B)$ . However,  $C_{P, \mathcal{L}}(B)$  may contain more than one option. These are the (perhaps multiple) options that the law abiding citizen prefers among the legal ones. Consider the following example: A preference  $P^*$  is indifferent between  $x$ ,  $y$ , and  $z$ . Assume no law and so,  $\mathcal{L}^*(B) = B$  for every issue  $B$ . Then,  $C^*$  ( $= C_{P^*, \mathcal{L}^*}$ ) is such that

$$C^*({x, y}) = {x, y}, \quad C^*({y, z}) = {y, z} \quad \text{and} \quad C^*({x, z}) = {x, z}.$$

Then, a choice correspondence where all options are selected is permitted, even in the case of a standard economic agent that optimizes a preference  $P$  on  $B$ .



Now consider the choice correspondence  $\bar{C}$ ,

$$\bar{C}(\{x, y\}) = \{x\}, \quad \bar{C}(\{y, z\}) = \{y, z\}, \quad \text{and} \quad \bar{C}(\{x, z\}) = \{x, z\}.$$

This choice correspondence is not possible for a standard economic agent. In the absence of any legal restriction,  $\bar{C}(\{x, y\}) = \{x\}$  implies a strict preference for  $x$  over  $y$ , while  $\bar{C}(\{y, z\}) = \{y, z\}$  and  $\bar{C}(\{x, z\}) = \{x, z\}$  implies indifference between  $x$ ,  $y$ , and  $z$ . This is an example of a choice correspondence that we refer to as non-spuriously cyclical. More generally,

**Definition 10** *A choice correspondence  $C$  is **non-spuriously cyclical** if there exist three distinct alternatives  $x$ ,  $y$  and  $z$  such that*

$$C(\{x, y\}) = \{x\}, \quad y \in C(\{y, z\}), \quad \text{and} \quad z \in C(\{x, z\}).$$

Non-spuriously cyclical choice correspondences are those that may induce cycles and cannot be produced by optimal choice when there is no law. Naturally, to be (or not to be) non-spuriously cyclical is a property of the choice correspondence itself. It is not a property of the final selection that might be made among optimal options.

**Definition 11** *A legal system  $\mathcal{L}$  **induces non-spurious cyclical choice correspondences** if there exists a preference  $P$  such that the resulting choice correspondence  $C_{P, \mathcal{L}}$  of a law abiding citizen is non-spuriously cyclical.*

We again speak of a legal system *inducing* non-spurious cyclic choice correspondences because they are not possible if there is no law and all options are legal.

**Legal Cycling Theorem (with possible indifferences)** Consider the case where issues are binary choices and there are at least three distinct alternatives. Then,

- (a) No option-stratified legal system induces non-spurious cyclic choice correspondences.

However,

- (b) Any legal system that is not option-stratified induces non-spurious cyclic choice correspondences.

**Proof:** See Appendix.

The Legal Cycling Theorem (with possible indifferences) characterizes the legal systems that induce non-spurious cyclic choice correspondences. In this variation of the Legal Cycling Theorem, the law-abiding citizen may be indifferent between options.

In the Combination Theorem the law itself might be cyclic (i.e.,  $CL$  holds). Then, the choices of a law abiding citizen are cyclic regardless of preferences. In particular, it does not matter whether the law abiding citizen may or may not be indifferent between options.

Finally, the Legal Cycling Theorem can also be modified to accommodate indifference and choices involving two or more options, but cycles must be replaced with the more general condition regarding WARP violations.

**Definition 12** *A choice correspondence  $C$  non-spuriously violates WARP if there exist two issues  $B$  and  $B^*$  and an option  $y$  such that*

$$B \subseteq B^*, y \in C(B^*) \cap B \text{ and } y \notin C(B).$$

This definition is the counterpart of WARP violations for correspondences. Any choice function that violates WARP is also a *choice correspondence that non-spuriously violates WARP*.

**Definition 13** *A full legal system  $\mathcal{L}$  induces choice correspondences that non-spuriously violate WARP if there exists a preference  $P$  such that the resulting choice correspondence of a law abiding citizen  $C_{P,\mathcal{L}}$  non-spuriously violates WARP.*

That is, consider a full legal system that induces choice correspondences that non-spuriously violate WARP. They produce choice correspondences that do not arise in the absence of law.

**Extended Legal Cycling Theorem (with possible indifferences)** Consider the case where there are at least three distinct alternatives. Then,

- (a) No full legal system that is option-stratified induces choice correspondences that non-spuriously violate WARP.

However,

- (b) Any full legal system that is not option-stratified induces choice correspondences that non-spuriously violate WARP.

**Proof:** See Appendix.

This result shows that even if the law-abiding citizen may be indifferent between options, the types of choice correspondence that may occur in the absence of any law (i.e., by standard economic agents) are the ones that arise under option-stratified full legal systems and only under them. If the full legal system is not option-stratified, it induces correspondences with a logical structure that do not arise in the absence of law.

## 10 A Lingering Question: Context-Dependent Alternatives

There is a question that repeatedly arises when certain choices are described as intransitive. In closely looking at a given case of intransitivity, one starts to wonder whether the intransitivity might not be spurious, whether it could not be made to go away if we are only careful enough about describing the alternatives before us.

Consider again our case of self-defense. We imagined a decision-maker who is willing to incur serious injury to protect a manuscript from great harm. We also noted that the law of self-defense will allow him to protect his body from great harm by using deadly force, but will not allow him to do so to protect his manuscript from destruction. This meant that when choosing between inflicting deadly harm on someone or suffering great harm himself, he will find himself choosing the former; that when choosing between suffering great harm to himself or to his manuscript, he will once again find himself choosing the former; but that when choosing between inflicting great harm on someone else or suffering great harm to his manuscript, he will choose the latter, thus resulting in a cycle. What if one were to distinguish between inflicting great harm on another for the sake of protecting one's manuscript, and inflicting great harm on another for the sake of protecting one's body, in other words, if one were to distinguish between inflicting great harm on another legally and inflicting such harm illegally? Having thus split what appears to be a single alternative into two, has not the intransitivity now been made to disappear? Something analogous could be tried with every one of our examples.

There are several difficulties with this approach. The most immediate worry is the one we noted before, namely that it proves too much. We have here a strategy that could be used to make all intransitivities go away, that is to say not only in our examples, but in all cases. But do we really want to deny the possibility of intransitivity altogether?

A difficulty that goes more to the heart of the matter, however, is one that surfaced when this possibility was first explored in the early days of decision theory. What became clear rather quickly, then, was that if one does not insist that alternatives be independent of their context, independent, that is, of the other alternatives in the choice set, various kinds of unpalatable logical consequences start to abound. In other words, the meaning of an option  $x$  cannot be allowed to change depending on whether  $y$  or  $z$  is also available. This assumption is implicit in almost every formal model and in this one as well. As we now show, if this assumption is relaxed, then the decision maker cannot order all alternatives because some choices would be impossible to make.

To see this, recall that an order is a complete, transitive binary relation. These are

the two traditional pillars of rationality. That is, completeness and transitivity are often equated with rationality (see definition 1.B.1, chapter 1 in *Microeconomic Theory*; Mas Colell, Whiston, and Green 1995). Completeness requires some decision to be possible between any two alternatives. Now for concreteness consider option  $x$  (to use deadly force) in our self-defense example in section 3.3. Let's break option  $x$  into two new alternatives:  $x_l$  and  $x_i$ . Option  $x_l$  is to use deadly force legally (i.e., in legitimate self-defense)). Option  $x_i$  is to use deadly force illegally. In this example, the use of deadly force is legal if the alternative is to incur a serious injury ( $y$ ), but not if it is to have the manuscript damaged ( $z$ ). Thus, it is impossible to make a choice between  $x_l$  and  $z$ . It is also impossible to make a choice between  $x_i$  and  $y$  for the same reason. If  $z$  is the available alternative then the use of deadly force is illegal. Hence, one cannot make a choice between  $x_l$  and  $z$  because  $x_l$  does not exist in the presence of  $z$ . Naturally, this is a general phenomenon that does not hinge on the specific example concerning self-defense. If an option ceases to exist in the presence of another option, then completeness no longer holds and, therefore, the logical structure of the choice function is not as in an order and, hence, it is fundamentally different from the ones in traditional choice theory that abstracts away from legal constraints. This limitation, however, does not apply to context-independent contingencies. For example "taking an umbrella if it rains" is context-independent because whether or not it rains does not depend on the available alternatives. Context-dependent contingencies on the other hand (such as legality) make choices incompatible with orders. Thus, our broad claim that legal constraints lead to choices incompatible with orders still hold even under the extremely unusual approach of allowing for such contingencies.

Finally, there is a very practical difficulty with this approach to intransitivity. Intransitivities are worrisome because they open us up to exploitative actions. In law, the most important kind of opportunity is the possibility of getting to a forbidden end by choosing an indirect path, such as the one we described in connection with our duress example: someone unable to protect his manuscript by participating in a crime, arranged things so that he could indirectly achieve that very trade-off by making use of the intransitivity. For each of our examples, such an arbitrage-like strategy could be constructed. Consider again the case of necessity, in which a cycle resulted from the fact that the decision-maker is permitted to break into a cabin once he is stranded without a safe means of descent, but not if he can simply abstain from the climb. The intransitivity allows him to first climb the mountain and then break into the cabin, rather than first break into the cabin and then climb the mountain. Such arbitrage opportunities are, as it were, the practical manifestations of the presence of a genuine intransitivity and cannot be made to go away by the conceptual magic of splitting a single alternative into two different context-dependent alternatives.

## 11 Conclusion

A legal system is option-stratified if it is possible to rank-order all legal options a citizen might face, and if the system requires that he chooses the highest ranked alternative among the options available to him. We show that an option-stratified system is the only one that can avoid cycling. We then show, through suitably representative examples and one general proposition, why no acceptable legal system is going to be option-stratified, and why all acceptable systems are therefore bound to induce cycles. Several implications of this result remain to be explored.

## 12 Appendix

### 12.1 Proof of the Legal Cycling Theorem (and Related results)

First consider the basic case where issues are binary choices and preferences are asymmetrical orders. That is, we first demonstrate the legal cycling theorem.

Assume that  $\mathcal{L}$  is an option-stratified legal system. Also assume, by contradiction, that there is an asymmetric preference order  $P$  such that, for the resulting choice function  $C$  ( $= C_{P,\mathcal{L}}$ ), there are distinct alternatives  $x, y$  and  $z$  such that  $C(\{x, y\}) = x$ ;  $C(\{y, z\}) = y$ ;  $C(\{x, z\}) = z$ . Then,  $x \in \mathcal{L}(\{x, y\})$ ,  $y \in \mathcal{L}(\{y, z\})$ , and  $z \in \mathcal{L}(\{x, z\})$ . So,  $u(x) \geq u(y) \geq u(z) \geq u(x)$ . Thus,  $u(x) = u(y) = u(z)$ . It now follows that  $\mathcal{L}(\{x, y\}) = \{x, y\}$ ,  $\mathcal{L}(\{y, z\}) = \{y, z\}$ , and  $\mathcal{L}(\{x, z\}) = \{x, z\}$ . Thus,  $x P y$  and  $y P z$  and  $z P x$ . A contradiction (with the transitivity of  $P$ ).

Now for the converse. Assume, by contradiction, that  $\mathcal{L}$  is a legal system that is not option-stratified and, for no asymmetric preference order  $P$ , the resulting choice function  $C$  ( $= C_{P,\mathcal{L}}$ ) is cyclical.

Step 1. There cannot be three distinct options  $x, y$  and  $z$  such that for two different pairs of options, say  $\{x, y\}$  and  $\{y, z\}$ , the law allows both choices, e.g.,  $\mathcal{L}(\{x, y\}) = \{x, y\}$  and  $\mathcal{L}(\{y, z\}) = \{y, z\}$ , and for the remaining pair  $\{x, z\}$ ,  $\mathcal{L}(\{x, z\})$  has only one element.

Assume that  $\mathcal{L}(\{x, z\}) = \{z\}$ . Consider any asymmetric preference order such that  $x P y P z$ . Then,  $C(\{x, y\}) = x$ ;  $C(\{y, z\}) = y$ ;  $C(\{x, z\}) = z$ . Now assume that  $\mathcal{L}(\{x, z\}) = \{x\}$ . Consider any asymmetric preference order such that  $z P y P x$ . Then,  $C(\{z, y\}) = z$ ;  $C(\{y, x\}) = y$ ;  $C(\{x, z\}) = x$ .

Let  $\succ$  be the binary relation defined by

$$x \succ y \Leftrightarrow \mathcal{L}(\{x, y\}) = \{x\}.$$

Step 2  $\succ$  is transitive.

Assume that  $x \succ y \succ z$ . If  $\mathcal{L}(\{x, z\}) = \{z\}$  then, for any preference  $P$ , the choices of a law abiding citizen are cyclical (because the law requires  $x$  to be chosen over,  $y$  over  $z$  and  $z$  over  $x$ ). If  $\mathcal{L}(\{x, z\}) = \{x, z\}$  then consider any asymmetric preference order  $P$  such that  $z P x$ , the choices of a law abiding citizen are cyclical. Thus,  $x \succ z$ .

Let  $S$  a chain be a sequence of options  $x_n, \dots, x_1$  such that  $x_j \succ x_i$  if  $j > i$ . Such a chain must exist, otherwise all options are legal and so,  $\mathcal{L}$  is an option-stratified legal system. Moreover, given that  $A$  is finite, there must exist a longest chain (one for which the number of elements in it is maximal). With some abuse of notation, let  $S = [x_n, \dots, x_1]$  be a longest, not necessarily unique chain. By definition, there is no option  $x$  such that  $x \succ x_n$  and no option  $y$  such that  $x_1 \succ y$ .

Step 3 For any alternative  $y$  that does not belong to the chain  $S$ , there exists a unique element  $x_i \in S$  such that  $\mathcal{L}(\{y, x_i\}) = \{y, x_i\}$ .

Assume that there are two distinct elements,  $x_i \in S$  and  $x_j \in S$  such that  $\mathcal{L}(\{y, x_i\}) = \{y, x_i\}$  and  $\mathcal{L}(\{y, x_j\}) = \{y, x_j\}$ . By definition,  $\mathcal{L}(\{x_i, x_j\})$  has only one element. This contradicts step 1. Now assume that for some alternative  $y$  there is no element  $x_i$  in chain  $S$  such that  $\mathcal{L}(\{y, x_i\}) = \{y, x_i\}$ . Thus,  $x_n \succ y$  and  $y \succ x_1$ . The chain  $S$  must contain more than 2 options. Otherwise the chain  $x_n \succ y \succ x_1$  is longer. Moreover, for any  $i = 2, \dots, n$ , if  $x_i \succ y$  then  $x_{i-1} \succ y$ . Otherwise  $x_i \succ y \succ x_{i-1}$  and so, the chain  $x_n \succ \dots x_i \succ y \succ x_{i-1} \succ \dots \succ x_1$  is longer. It follows that either  $x_1 \succ y$  or  $y \succ x_n$ . A contradiction.

Given any alternative  $y$  not in the chain  $S$ , let  $i(y) \in \{1, \dots, n\}$  be such that  $\mathcal{L}(\{y, x_{i(y)}\}) = \{y, x_{i(y)}\}$ . If  $z$  is in the chain  $S$  let  $i(z) \in \{1, \dots, n\}$  be such that  $z = x_{i(z)}$ . For any alternative  $x$ , let  $u(x) = i(x)$ . By step 3,  $u$  is well defined.

Step 4. If an option  $y$  is not in the chain  $S$  then  $\mathcal{L}(\{y, x_j\}) = \{y\}$  if  $j < i(y)$  and  $\mathcal{L}(\{y, x_j\}) = \{x_j\}$  if  $j > i(y)$ .

The case  $\mathcal{L}(\{y, x_j\}) = \{y, x_j\}$  can be ruled out by step 1 because  $\mathcal{L}(\{x_j, x_{i(y)}\})$  has only one element and, by definition,  $\mathcal{L}(\{y, x_{i(y)}\}) = \{y, x_{i(y)}\}$ . Let  $x_i = x_{i(y)}$ . Assume that  $j < i(y)$ . Then,  $C(\{x_i, x_j\}) = x_i$ . Consider any asymmetric order  $P$  such that  $y P x_i$ . Then,  $C(\{x_i, y\}) = y$ . If  $\mathcal{L}(\{y, x_j\}) = \{x_j\}$  then  $C(\{y, x_j\}) = \{x_j\}$ . This forms a cycle. So,  $\mathcal{L}(\{y, x_j\}) = \{y\}$ . Now assume that  $j > i$ . Then,  $C(\{x_i, x_j\}) = x_j$ . Consider any order  $P$  such that  $x_i P y$ . Then,  $C(\{x_i, y\}) = x_i$ . If  $\mathcal{L}(\{y, x_j\}) = \{y\}$  then  $C(y, x_j) = y$ . This forms a cycle. So,  $\mathcal{L}(\{y, x_j\}) = x_j$ .

Step 5. If  $z$  and  $y$  are distinct options and neither is in  $S$  then  $\mathcal{L}(\{y, z\}) = z$  if  $i(y) < i(z)$ ;  $\mathcal{L}(\{y, z\}) = y$  if  $i(y) > i(z)$  and  $\mathcal{L}(\{y, z\}) = \{y, z\}$  if  $i(y) = i(z)$ .

Consider the case  $i(y) < i(z)$ . By step 4,  $\mathcal{L}(\{z, x_{i(y)}\}) = z$ . By definition,  $\mathcal{L}(\{y, x_{i(y)}\}) = \{y, x_{i(y)}\}$ . By step 1, the case  $\mathcal{L}(\{y, z\}) = (y, z)$  can be ruled out. Now consider the case  $\mathcal{L}(\{y, z\}) = y$ . Then,  $C(\{y, z\}) = y$ . Given that  $\mathcal{L}(\{z, x_{i(y)}\}) = z$  then  $C(\{z, x_{i(y)}\}) = z$ .

Now consider any order  $P$  such that  $y P x_{i(y)}$ . By definition,  $\mathcal{L}(\{y, x_{i(y)}\}) = \{y, x_{i(y)}\}$ . So,  $C(\{y, x_{i(y)}\}) = y$ . This forms a cycle. Thus,  $\mathcal{L}(\{y, z\}) = z$ . The proof on the case  $i(y) > i(z)$  is the same with just a change in labels. Hence, it is omitted. Now consider the case  $i(y) = i(z)$ . Let  $x_i = x_{i(y)} = x_{i(z)}$ . By definition,  $\mathcal{L}(\{y, x_i\}) = \{y, x_i\}$  and  $\mathcal{L}(\{z, x_i\}) = \{z, x_i\}$ . So, by step 1,  $\mathcal{L}(\{y, z\}) = \{y, z\}$ .

The proof is now concluded as follows: Let  $x$  and  $y$  be the distinct options. If both of them belong to the chain  $S$  then, by definition,  $u(x) \neq u(y)$  and  $\mathcal{L}(\{x, y\}) = x$  if  $u(x) > u(y)$ . If one of them, say  $x$  belongs to the chain  $S$ , and the other,  $y$ , does not then, by step 4,  $\mathcal{L}(\{x, y\}) = x$  if  $u(x) > u(y)$  and  $\mathcal{L}(\{x, y\}) = y$  if  $u(y) > u(x)$ . By definition,  $\mathcal{L}(\{x, y\}) = \{x, y\}$  if  $u(y) = u(x)$ . If both  $x$  and  $y$  do not belong to  $S$  then, by step 5,  $\mathcal{L}(\{x, y\}) = \{x\}$  if  $u(x) > u(y)$ ,  $\mathcal{L}(\{x, y\}) = \{y\}$  if  $u(y) > u(x)$  and  $\mathcal{L}(\{x, y\}) = \{x, y\}$  if  $u(y) = u(x)$ . Therefore,  $\mathcal{L}$  is an option-stratified system. A contradiction. This demonstrates the legal cycling theorem.

Now we consider the general case where issues can have two or more alternatives in it. So, we now demonstrate the extended legal cycling theorem.

Assume that  $\mathcal{L}$  is a full option-stratified legal system. Also assume, by contradiction, that there is an asymmetric preference order  $P$  such that, for the resulting choice function  $C (= C_{P, \mathcal{L}})$ , there are issues  $B$  and  $B^*$  such that  $B \subseteq B^*$ ,  $C(B^*) \in B$  and  $C(B) \neq C(B^*)$ . Then,  $u(C(B)) \geq u(C(B^*))$  (because  $C(B^*) \in B$ ) and  $u(C(B^*)) \geq u(C(B))$  (because  $C(B) \in B^*$ ). Thus,  $u(C(B)) = u(C(B^*))$ . Therefore,  $C(B) \in \mathcal{L}(B^*)$  and  $C(B^*) \in \mathcal{L}(B)$ . It follows that  $C(B) P C(B^*)$  and  $C(B^*) P C(B)$ . A contradiction.

Now for the converse. Assume, by contradiction, that  $\mathcal{L}$  is a full legal system that is not option-stratified and, for no asymmetric preference order  $P$ , the resulting choice function  $C (= C_{P, \mathcal{L}})$  violates WARP. Then, in particular, no resulting choice function  $C$  is cyclical. Hence, by the argument above, the utility function  $u$  is well defined and such that

$$\mathcal{L}(\{x, y\}) = \{x, y\} \text{ if } u(x) = u(y); \mathcal{L}(\{x, y\}) = \{x\} \text{ if } u(x) > u(y).$$

Now assume that  $x \in \mathcal{L}(B)$  and  $u(y) > u(x)$  for some  $y \in B$ . Then,  $\mathcal{L}(\{x, y\}) = y$ . Let  $P$  be any preference order such that  $x P z$  for any  $z \neq x$ . Then,  $C(B) = x$  and  $C(x, y) = y$ . Thus,  $C$  violates WARP. A contradiction. Now assume that  $u(x) \geq u(y)$  for every  $y \in B$  and  $x \notin \mathcal{L}(B)$ . Then,  $C(B) \neq x$  and  $u(x) \geq u(C(B))$ . Let  $z = C(B)$ . So,  $x \in \mathcal{L}(\{z, x\})$ . Let  $P$  be any asymmetric preference order such that  $x P z$ . Then,  $x = C(x, z)$ ,  $x \in B$  and  $z = C(B) \neq x$ . Thus,  $C$  violates WARP. A contradiction.

Now we demonstrate the legal cycling theorem (with possible indifferences). Assume that  $\mathcal{L}$  is an option-stratified legal system. Also assume, by contradiction, that there is a preference order  $P$  such that, for the resulting choice function  $C (= C_{P, \mathcal{L}})$ , there are distinct

alternatives  $x$ ,  $y$  and  $z$  such that

$$C(\{x, y\}) = \{x\}; y \in C(\{y, z\}); z \in C(\{x, z\}).$$

Then,  $x \in \mathcal{L}(\{x, y\})$ ,  $y \in \mathcal{L}(\{y, z\})$ , and  $z \in \mathcal{L}(\{x, z\})$ . So,  $u(x) \geq u(y) \geq u(z) \geq u(x)$ . Thus,  $u(x) = u(y) = u(z)$ . It now follows that  $\mathcal{L}(\{x, y\}) = \{x, y\}$ ,  $\mathcal{L}(\{y, z\}) = \{y, z\}$ , and  $\mathcal{L}(\{x, z\}) = \{x, z\}$ . Thus,  $x P y$  and  $y P z$  and  $z P x$ . By transitivity,  $x P y$  and  $y P x$ . Therefore,  $C(\{x, y\}) = \{x, y\}$ . A contradiction.

For the converse. If  $\mathcal{L}$  is not an option-stratified legal system then there is a preference order (asymmetric)  $P$  such that the resulting choice function  $C (= C_{P, \mathcal{L}})$  is cyclic. Therefore,  $C(\{x, y\}) = x$ ;  $C(\{y, z\}) = y$ ;  $C(\{x, z\}) = z$ . It follows that  $C$  is non-spuriously cyclic.

Finally, we demonstrate the extended legal cycling theorem (with possible indifferences). Assume that  $\mathcal{L}$  is a full option-stratified legal system. Also assume, by contradiction, that there is a preference order  $P$  such that, for the resulting choice function  $C (= C_{P, \mathcal{L}})$ , there are issues  $B$  and  $B^*$  and an option  $y$  such that

$$B \subseteq B^*, y \in C(B^*) \cap B \text{ and } y \notin C(B).$$

Let  $z \in \mathcal{L}(B)$ . Then,  $u(z) \geq u(y)$  (because  $y \in B$ ) and  $u(y) \geq u(z)$  (because  $y \in \mathcal{L}(B^*)$  and  $z \in B^*$  given that  $B^* \supseteq B \supseteq \mathcal{L}(B)$ ). Thus,  $u(y) = u(z)$ . Therefore,  $z \in \mathcal{L}(B^*)$ . It follows that  $y P z$ . This holds for any  $z \in \mathcal{L}(B)$ . Hence,  $y \in C(B)$ . A contradiction.

For the converse. If  $\mathcal{L}$  is a full legal system that is not option-stratified then there is a preference order (asymmetric)  $P$  such that the resulting choice function  $C (= C_{P, \mathcal{L}})$  violates WARP. Thus,  $C$  non-spuriously violates WARP.

## 12.2 Proof of the Proposition

First consider the case  $n = 2$ . Let  $x$ ,  $y$  and  $z$  be three different choices. Let  $u_1 : A \rightarrow \mathcal{R}$  be any function such as  $u_1(y) = u_1(z) > u_1(x)$ . Let  $u_2 : A \rightarrow \mathcal{R}$  be an function such as  $u_2(y) = u_2(z) = u_2(x)$ . Let  $\mathcal{D}_1$  be any doctrine such that

$$\mathcal{D}_1(\{y, z\}) = \{y, z\}; \mathcal{D}_1(\{x, y\}) = n/a; \mathcal{D}_1(\{x, z\}) = \{z\}.$$

and for any other issue  $B = \{w, v\}$ , where either  $w \notin \{x, y, z\}$  or  $v \notin \{x, y, z\}$  or both  $w$  and  $v$  do not belong to  $\{x, y, z\}$ ,

$$\text{either } \mathcal{D}_1(\{w, v\}) = n/a \text{ or } \mathcal{D}_1(\{w, v\}) = \{w, v\} \text{ if } u_1(w) = u_1(v);$$

$$\text{either } \mathcal{D}_1(\{w, v\}) = n/a \text{ or } \mathcal{D}_1(\{w, v\}) = \{w\} \text{ if } u_1(w) > u_1(v);$$

$$\text{either } \mathcal{D}_1(\{w, v\}) = n/a \text{ or } \mathcal{D}_1(\{w, v\}) = \{v\} \text{ if } u_1(v) > u_1(w).$$



Let  $\mathcal{D}_2$  be any doctrine such that

$$\mathcal{D}_2(y, z) = (y, z); \mathcal{D}_1(x, y) = (x, y); \mathcal{D}_1(x, z) = n/a$$

and for any other issue  $B = (w, v)$ , where either  $w \notin \{x, y, z\}$  or  $v \notin \{x, y, z\}$  or both  $w$  and  $v$  do not belong to  $\{x, y, z\}$ ,

$$\text{either } \mathcal{D}_2(\{w, v\}) = n/a \text{ or } \mathcal{D}_2(\{w, v\}) = \{w, v\} \text{ if } u_2(w) = u_2(v);$$

$$\text{either } \mathcal{D}_2(\{w, v\}) = n/a \text{ or } \mathcal{D}_2(\{w, v\}) = \{w\} \text{ if } u_2(w) > u_2(v);$$

$$\text{either } \mathcal{D}_2(\{w, v\}) = n/a \text{ or } \mathcal{D}_2(w, v) = \{v\} \text{ if } u_2(v) > u_2(w).$$

By construction,  $\mathcal{D}_1$  and  $\mathcal{D}_2$  are conditional option-stratified doctrines. Let  $\alpha$  be an aggregator that maps conditional option-stratified doctrines into option-stratified legal systems and  $\mathcal{L} = \alpha(\mathcal{D}_1, \mathcal{D}_2)$ . By unanimity,

$$\mathcal{L}(\{y, z\}) = \{y, z\}; \mathcal{L}(\{x, y\}) = \{x, y\}; \mathcal{L}(\{x, z\}) = \{z\}.$$

Thus,  $\mathcal{L}$  is not option-stratified. A contradiction.

The case  $n > 2$  can be shown in exactly the same way.

The argument above shows our result in the case that  $\mathcal{L}$  is not option-stratified because  $(CD)$  holds. A proof where  $\mathcal{L}$  is not option-stratified because  $(CL)$  holds can also be obtained. Again we focus on the case  $n = 2$ .

Let  $u_1 : A \rightarrow \mathcal{R}$  be any function such as  $u_1(x) > u_1(y) > u_1(z)$ . Let  $u_2 : A \rightarrow \mathcal{R}$  be an function such as  $u_2(z) > u_2(x)$ . Let  $\mathcal{D}_1$  be any doctrine such that

$$\mathcal{D}_1(\{x, y\}) = \{x\}; \mathcal{D}_1(\{y, z\}) = \{y\}; \mathcal{D}_1(\{x, z\}) = n/a$$

and for any other issue  $B = (w, v)$ , where either  $w \notin \{x, y, z\}$  or  $v \notin \{x, y, z\}$  or both  $w$  and  $v$  do not belong to  $\{x, y, z\}$ ,

$$\text{either } \mathcal{D}_1(\{w, v\}) = n/a \text{ or } \mathcal{D}_1(\{w, v\}) = \{w, v\} \text{ if } u_1(w) = u_1(v);$$

$$\text{either } \mathcal{D}_1(\{w, v\}) = n/a \text{ or } \mathcal{D}_1(\{w, v\}) = \{w\} \text{ if } u_1(w) > u_1(v);$$

$$\text{either } \mathcal{D}_1(\{w, v\}) = n/a \text{ or } \mathcal{D}_1(\{w, v\}) = \{v\} \text{ if } u_1(v) > u_1(w).$$

Let  $\mathcal{D}_2$  be any doctrine such that

$$\mathcal{D}_1(\{x, y\}) = n/a; \mathcal{D}_2(\{y, z\}) = n/a; \mathcal{D}_1(\{x, z\}) = \{z\}.$$

and for any other issue  $B = \{w, v\}$ , where either  $w \notin \{x, y, z\}$  or  $v \notin \{x, y, z\}$  or both  $w$  and  $v$  do not belong to  $\{x, y, z\}$ ,

$$\text{either } \mathcal{D}_2(\{w, v\}) = n/a \text{ or } \mathcal{D}_2(\{w, v\}) = \{w, v\} \text{ if } u_2(w) = u_2(v);$$

either  $\mathcal{D}_2(\{w, v\}) = n/a$  or  $\mathcal{D}_2(\{w, v\}) = \{w\}$  if  $u_2(w) > u_2(v)$ ;  
 either  $\mathcal{D}_2(\{w, v\}) = n/a$  or  $\mathcal{D}_2(\{w, v\}) = \{v\}$  if  $u_2(v) > u_2(w)$ .

By construction,  $\mathcal{D}_1$  and  $\mathcal{D}_2$  are conditional option-stratified doctrines. Let  $\alpha$  be an aggregator that maps conditional option-stratified doctrines into option-stratified legal systems and  $\mathcal{L} = \alpha(\mathcal{D}_1, \mathcal{D}_2)$ . By unanimity,

$$\mathcal{L}(\{x, y\}) = \{x\}; \mathcal{L}(\{y, z\}) = \{y\}; \mathcal{L}(\{x, z\}) = \{z\}.$$

Thus,  $\mathcal{L}$  is not option-stratified.

## References

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